ELEC 531 HW 9

Due Wednesday, Nov. 3, beginning of class

1. ROC Curves (10 points)
   For the two detection problems considered in the previous homework, plot the ROCs of the corresponding Neyman-Pearson detectors. Recall that the ROCs should be concave and above the “random guessing” line. For the Poisson problem, use \( N = 20 \).

2. Colored noise (15 points; 3 points each)
   Consider a binary communication system were we send a 1 by transmitting \( s_1 = [1/2\ 1/2]^T \), and we send a 0 by transmitting \( s_0 = [-1/2\ -1/2]^T \). We observe \( x = s_k + w \), where \( w \) is colored Gaussian noise:
   \[
   w \sim \mathcal{N}(0, R),
   \]
   with
   \[
   R = \begin{bmatrix}
   1 & \rho \\
   \rho & 1
   \end{bmatrix},
   \]
   \(-1 < \rho < 1\). In other words, \( \mathbb{E}[w(0)^2] = \mathbb{E}[w(1)^2] = 1 \) and \( \mathbb{E}[w(0)w(1)] = \rho \).
   
   a. Reduce the likelihood ratio test to a test involving a one-dimensional statistic.
   
   b. Determine the distribution of the test statistic under each hypothesis.
   
   c. Write the false alarm and detection probabilities in terms of the threshold \( \gamma \) (from the simplified test) and \( \rho \).
   
   d. Write \( P_D \) as a function of \( P_F \) and \( \rho \). Identify the SNR.
   
   e. Plot (on one set of axes) the ROC for several different values of the correlation coefficient \( \rho \in (-1,1) \). When is detection in colored noise easier than detection in white noise \((\rho = 0)\)? What happens when \( \rho \to 1 \), or \( \rho \to -1 \)?