

(2) ALTERNATE  
SOLUTIONS

to HW 6. PROB. 1

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$$Z = |X - Y|, \quad \begin{array}{l} X \sim U(0,1) \\ Y \sim U(0,1) \end{array}$$

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$$P_Z(z) = ?$$

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$$\underline{Z = |X - Y|}$$

$$X \sim U(0,1)$$

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$$\underline{Y \sim U(0,1)}$$

$$P_Z(z) = ?$$

APPROACH (1)

VIA CDF

$$F_Z(z) = P[Z \leq z] = P[|X - Y| \leq z]$$

$$= P[-z \leq \underbrace{X - Y}_W \leq z]$$

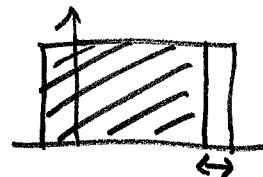
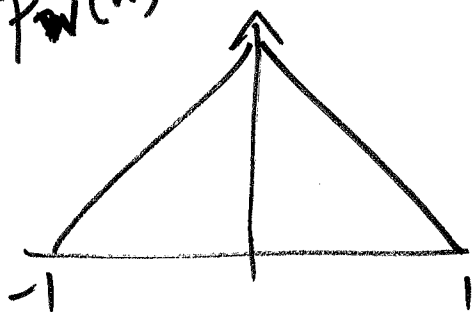
$$= P[W \leq z] - P[W \leq -z]$$

$$= \underline{F_W(z) - F_W(-z)}$$

$$\underline{W = X - Y}$$

$$P_W(w) = P_{X-Y}(w) = \int_{-\infty}^{\infty} P_X(x) P_Y(w-x) dx$$

$P_W(w)$



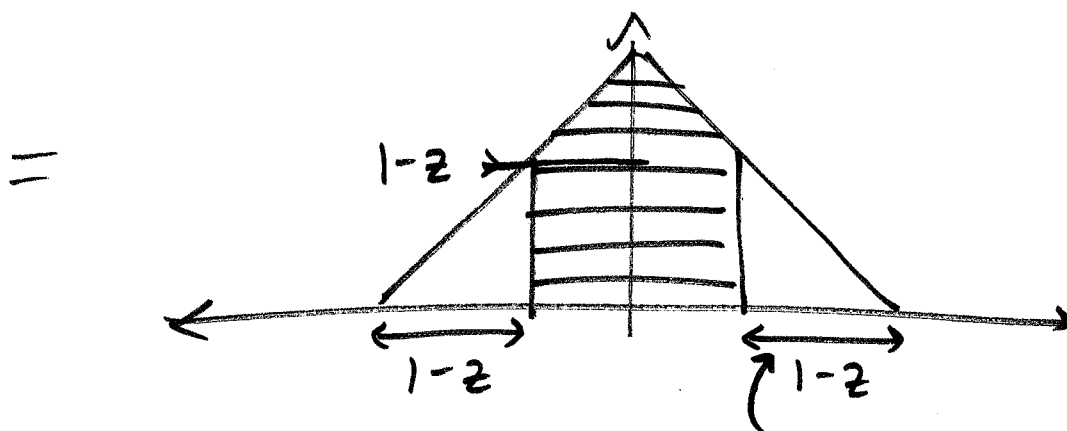
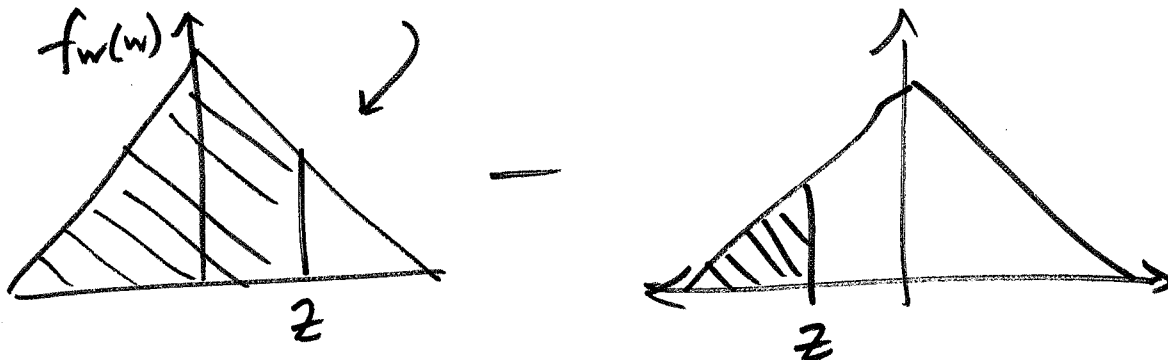
$$= \int P_X(x) P_Y(x-w) dx$$

convolve  
X w/ non-flipped  
inversion of Y.

For  $0 \leq z \leq 1$ .

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$$F_z(z) = F_w(z) - F_w(-z)$$



$$F_z(z) = 1 - 2 \left( \frac{\text{width}}{2} \right) \left( \frac{\text{ht.}}{2} \right)$$

$$= 1 - (1-z)(1-z)$$

$$= 1 - (1 - 2z + z^2) = 2z - z^2.$$

$$f_z(z) = \frac{d}{dz} F_z(z) = 2 - 2z = 2(1-z),$$

$0 \leq z \leq 1$

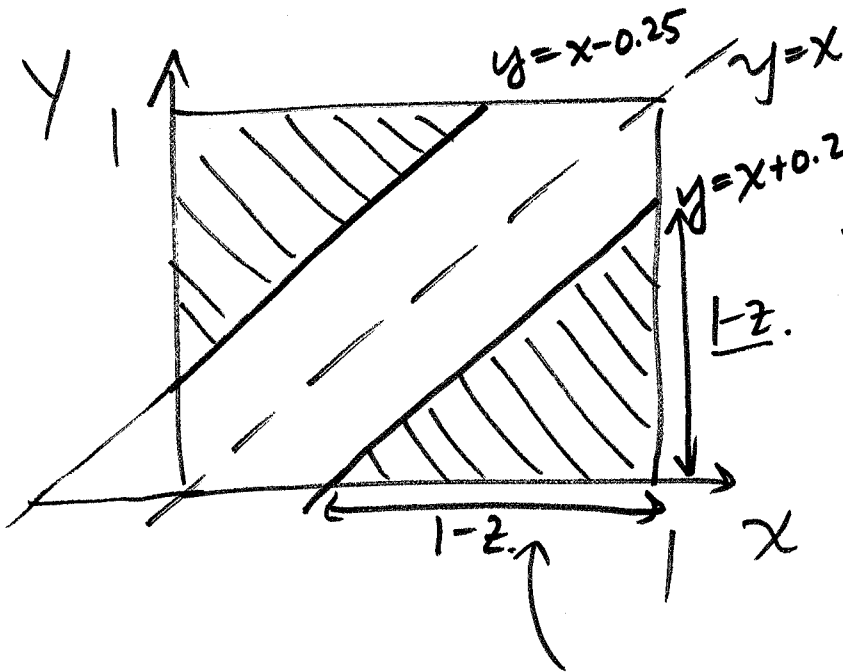
Alternatively,  $X \sim U(0,1), Y \sim U(0,1)$

domain

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$$Z = |X - Y|$$

$$0 \leq Z \leq 1$$



if  $z = 0.25$   
 $\Rightarrow x - y = \pm 0.25$   
 $\Rightarrow y = x - 0.25$   
 $x + 0.25$

$$F_Z(z) = P[Z \leq z] = P[|X - Y| \leq z]$$

$\Rightarrow$  For  $0 \leq z \leq 1$

$$F_Z(z) = 1 - 2 \left( \frac{1-z}{2} \right) (1-z)$$

$$= 1 - (1-z)(1-z)$$

SAME AS BEFORE !!