$VAR(X_1 + X_2) = ?$ X_1, X_2 Are Not assumed to be independent!

1/1

$= E[((X_1+X_2) - E[X_1+X_2])^2]$

$= E[(X_1+X_2)^2] - (E[X_1+X_2])^2$

$= E[X_1^2] + 2E[X_1 X_2] + E[X_2^2]$
$\quad - (E[X_1]^2 + 2E[X_1]E[X_2] + E[X_2]^2)$

$$= \underbrace{E[X_1^2] - E[X_1]^2}_{VAR(X_1)} + \underbrace{E[X_2^2] - E[X_2]^2}_{VAR(X_2)}$$
$$+ 2(\underbrace{E[X_1 X_2] - E[X_1]E[X_2]}_{cov(X_1, X_2) = E[(X_1 - E[X_1])(X_2 - E[X_2])]})$$

$= VAR(X_1) + VAR(X_2) + 2\,cov(X_1, X_2)$

For the sum of an arbitrary # of RVs,

$M = \sum_{i=1}^{N} X_i$,

$$VAR(M) = \sum_{i=1}^{N} VAR(X_i) + 2\sum_{i,j} cov(X_i, X_j)$$