A Flexible Framework for Polynomial-Time Resource Allocation in Multiflow Wireless Networks

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Network-Aware Resource Allocation

- Requires PHY parameter decisions jointly with MAC decisions
- Cross-layer design
- Large network
- Several information flows

**Challenge**

Systems must be implemented, which requires specified behavior at all times for all terminals
- Schedules
- Routes
- Power allocations

**Our Contribution**

Framework and methodology for specifying PHY and MAC behavior in large realistic wireless networks, in polynomial time
Example
Two Flows, Two Timeslots

Interference, not collision!
Fine-grained resource allocation entails:
- Schedules
- Routes
- Node behavior (ex. Tx power)

\[
\overline{T} = \frac{1}{2} \log(1 + \text{SINR}_B) + \frac{1}{2} \log(1 + \text{SINR}_Y)
\]
**General Problem**

**Resource Allocation Problem**

Choose schedules and routes $S$, power allocations $P$ at all timeslots $t$ so as to optimize some function of the network

$$\max_{S,P} f(S, P)$$

**Example Throughout this Talk**

Throughput: Maximize the minimum throughput $\eta$ across flows $f \in F$

$$\max_{S,P} \min_{F} \eta^f (S, P)$$

- $\eta^f$ is defined in terms of Shannon Capacity $\frac{1}{T} \log_2(1 + \text{SINR}_f)$
### Alternative Approaches

**Linear Programming**
- Relaxation
- Primal dual methods, worst-case exponential complexity
- Approximation problems

**Iterative Methods**
- Polynomial time link scheduling, topology-dependent order
- Lagrangian-multiplier update techniques

Efficient techniques which fully specify terminal behavior at all times in interfering networks are missing
System Setup

Assumption on the terminals:
- Half-duplex
- Transmit power $P_t(x) \in [0, P_{Tx}]$
- Rates are specified in terms of Shannon capacity $\Rightarrow$ interference, not collisions!

Assumption on the network: $F$ flows
- Distance-based pathloss model
- Only sources $S^f$ have traffic
- All other nodes have infinite buffers

Define a binary scheduling-routing variable $I^f_t(x, y)$:
- 1 if terminal $x$ is transmitting data to terminal $y$ at time $t$ for flow $f$.

![Diagram of terminal X to terminal Y]
Multiflow Constraints
Scheduling and Routing with Half-Duplex Terminals

\[
\sum_{x,y \in \mathcal{N}} I_t^f(x,y) = 1 \quad \forall f, t
\]

\[
\sum_{y \in \mathcal{N}} I_t^f(S^f, y) = 1 \quad \forall f
\]

\[
\sum_{f \in \mathcal{F}} I_t^f(x,y) \leq 1 \quad \forall t, (x,y)
\]

\[
\sum_{f \in \mathcal{F}} \sum_{x \in \mathcal{N}\setminus y} I_t^f(x,y) + \sum_{z \in \mathcal{N}\setminus y} I_t^f(y,z) \leq 1
\]

\[
\sum_{x \in \mathcal{N}} I_t^f(x,y) = \sum_{z \in \mathcal{N}} I_{t+1}^f(y,z) \quad \forall f, t
\]

\[
\sum_{x \in \mathcal{N}} I_T^f(x,D^f) = 1 \quad \forall f
\]

Integer Constraints:

- Half-Duplexing
- Route Endpoints
- Continuity and Conservation of Information

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Our Approach

Global Solution Unavailable
- Mixed-integer programming with non-convex continuous functions
- NP-Hard ⇒ Exhaustive search

Decomposition
- Flow-wise Decomposition
- Dynamic Programming from there

Complexity Management
Manage flow interactions:
Network-Flow Interaction Chart
Capture spatial and temporal interaction

- Nodes: terminals in network
- Edges: network action (e.g., transmission)
- Timeslots proceed horizontally
- Nodes and edges are weighted
Dynamic Programming
Chart the Progress of Information with Weights

Optimal Subproblem Property
Solutions exhibit regular behavior:
- Throughput: Bottleneck Link
- Delay: Minimum sum route

Example: Throughput
- Transmission limited: \( \min \)
- Choose best source: \( \max \)

The solution follows a **pattern**: at each timeslot, each terminal will always make an optimal choice.

Require the edge weights and update equations to form a **monotone semiring**

A single packet can be optimally scheduled \( \Rightarrow \) decompose by packets!
Flow Sequencing
Determine the Decomposition Sequence

Flow with most hops sets the schedule length

Spatial Reuse
- Schedule most resource-consuming flow first
- NFIC opportunistically schedules others “around” it

In the pathloss model, this corresponds to the longest flow first
Calculate Throughput:

\[ \eta^f = \frac{d^f_i}{t-1} \]

\[ \min\{d^i_1, e^{i,j}_1\} + S \max_j\{e^{i,j}_1\} \]
Suppose link between $X$ and $B$ is the bottleneck link ⇒ use full power at $X$.

- Scale power at $A$ such that $e_{1,A,X} = e_{2,X,B}$
Resource Subtraction

Enforcing Constraints on $I^f_t(x, y)$

- One packet per timeslot
- Half-duplex
- Interference
Interference Management

Resource Reuse

Fully interfering network $\Rightarrow$ rate loss

Manage interference, permit only $R_{\text{Int}}$ loss on existing packets

Edge Update

Edges represent link capacities—functions of transmission power $\Rightarrow$ modify them

At time $t$, for all nodes $x \in \mathcal{N}$

1. Calculate maximum power node $x$ can use so that interference on existing packets drops rates less than $R_{\text{Int}}$

2. Update all edges from node $x$ with new maximum rates
NFIC Resource Allocation in Action

Example Network

Terminals

- $S_1$
- $S_2$
- $S_3$

- $D_1$
- $D_2$
- $D_3$

waits for

to pass

is routed
left to avoid

Timeslots

1  2  3  4  5  6

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Complexity

\[ N(N - 1) + N \] edge and node updates per slot

\[ N \] slots (worst case)

\[ F \] flows

\[ \Rightarrow FN(N(N - 1) + N) = O(N^3) \]
Results I

Throughput Gains: Three Flows

Round Robin scheme: 1 hop per packet, 1 slot per flow.

\[ T_{\text{gain}} = \frac{\overline{T}_{\text{NFIC}} - \overline{T}_{\text{RR}}}{\overline{T}_{\text{RR}}} \]

- NFIC routing offers throughput gains
- Gains are much greater when high pathloss offers spatial isolation in the network
Results II

Power Savings: Three Flows

Round Robin scheme: 1 hop per packet, 1 slot per flow.

\[ \alpha = 4 \]

\[ \alpha = 3 \]

\[ \alpha = 2 \]

Relative Power Savings

Flows Routed

\[ \text{RPS} = \frac{P_{RR} - P_{NFIC}}{P_{NFIC}} \]

- NFIC efficiently introduces the multihop advantage
- Higher pathloss environments benefit considerably
Conclusions

- Proposed $O(N^3)$ framework for NP-hard resource allocation problems
- Specific throughput example
- Applicable to any monotone-semiring formulation

Future Work

- Study streaming packets
- Generalize nodes to incorporate cooperative links
- Study bounds on algorithmic performance

Thank you!

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