ELEC 306
Problem Set 8
Due: October 31, 2014

Homework Problems.
Work the following problems in Sadiku:

H8.1 10.40

H8.2 10.61

H8.3 10.67

H8.4 10.70

H8.5 Although Sadiku covers oblique reflection from the surface between two dielectrics, he neglects oblique reflection from a conductor, which is a simpler problem. Perhaps he intended to leave it as a homework problem, but he neglected to do that as well. We will remedy these oversights with the following problem:

A parallel polarized plane wave with electric field

$$\mathbf{E} = \mathbf{E_0}\mathrm{Re}[\mathbf{e}^{\mathbf{j}(\mathbf{k}\cdot\mathbf{r}-\omega\mathbf{t})}]$$

is incident on a flat perfectly conducting surface at an angle of incidence θ. For the reflected wave find: the direction of propagation, amplitude, phase, and polarization.

Real Problems.

R8.1 **Microwaving a hamburger.** The detailed physics of microwave cooking are sufficiently complex that at least one PhD thesis has been written about it. However, like most problems capable of being made arbitrarily complicated, this one can be make sufficiently simple to be solvable with bounded effort.

So let's start simplifying.

- A real microwave oven has a complex standing wave pattern caused by reflections from the walls. We will assume our hamburger is in a totally absorbent chamber (or free space) and irradiated by a uniform plane wave travelling in the $-z$ direction (from top to bottom).
- We will model the hamburger as a slab of uniform thickness h and diameter d, with $d \gg h$, lying on the x-y plane, centered at the origin.
- A real hamburger contains a large number of ingredients of widely varying properties. We will worry about only two of them, the bun and the hamburger patty. A bit of research yields the following values for the dielectric and thermal properties of hamburger ingredients at 2450 MHz, 300 K:

Material	$\epsilon_r{}'$	$\epsilon_r{}''$	k	ρ	C_p
			W m^{-1} K^{-1}	kg m^{-3}	J kg^{-1} K^{-1}
Bun	4.6	0.6	0.45	800	2850
Patty	43.0	15.0	0.49	1070	2510

Now let's get down to work:

(a) A useful tool for a simplified analysis is *Lambert's law* which states $P(z) = P_0 e^{-2\alpha z}$ where α is the attenuation coefficient and P_0 is the power flux at $z = 0$. Show that Lambert's law follows from the properties of EM wave propagation in a lossy dielectric.

(b) Lambert's law tells us how much power is *transmitted* into the hamburger, but the cooking is done by the power *absorbed* by the hamburger. Show that the power absorbed per unit volume is $p(z) = 2P_0 \alpha e^{-2\alpha z}$.

(c) For this part and the next, assume that we are cooking only the hamburger patty. We'll worry about the bun later. The hamburger patty is 1-cm thick and the power flux on the upper surface of the patty is $3\,\mathrm{W/cm^2}$ at a frequency of 2450 MHz. Plot the power absorbed (in $\mathrm{W/cm^3}$) vs depth into the patty.

(d) In some sense, the power absorbed per unit volume tells us how fast our hamburger is cooking, but a more conventional indicator would be how hot it is getting (i.e. its temperature). Given that we know the power density and the thermal properties of the patty, it is simple to find the temperature at all points. All we have to do is solve the *heat conduction equation*: $\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T + p(\mathbf{r})$ where T is the temperature, p is the absorbed power density, ρ is the density of the material, C_p its specific heat capacity, and k its thermal conductivity. However, this looks like a lot of work. Since we're supposed to be simplifying things, let's simplify this by assuming that the effect of thermal conductivity is small compared to the rate of power absorption. In this case we can neglect the first term on the right, leaving $\rho C_p \frac{\partial T}{\partial t} = p(\mathbf{r})$. Using this simplified model, plot T vs depth in the sample after one minute of cooking, assuming an ambient temperature of 300 K. What is the average temperature?

(e) Repeat parts (c) and (d) for the bun, assuming a thickness of 5 cm.

(f) Lambert's law assumes that the thickness of the slab is much larger than the penetration depth. If it isn't, we have to worry about reflections from the bottom surface of the slab. What is the penetration depth of our two ingredients?