Chapter 12

Inductive and Magnetic Sensors

12.1 Inductive Sensors

A number of the actuators developed in previous chapters depend on the variation of reluctance with changes in angle or displacement. Since $L = \frac{N^2}{\mathcal{R}}$ these changes in reluctance result in changes in inductance, allowing the same or similar sturctures to be used as sensors.

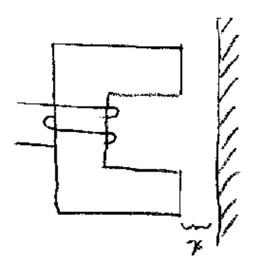
12.1.1 Measuring Inductance

Techniques for measuring inductance parallel those discussed in Chapter 5 for measuring capacitance.

1. Compare to a known inductor.
2. Connect to a known voltage and measure $\frac{di}{dt}$.
3. Measure the time constant when combined with a known resistor.
4. Measure the resonant frequency when combined with a known capacitor.

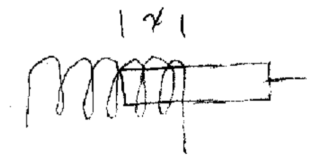
12.1.2 Proximity Sensors

We can use the variable gap electromagnet of Section 9.2.1 to measure the distance between the C-core and a ferromagnetic surface. Because of the $\frac{1}{x^2}$ behavior of the reluctance, this device is primarilly useful for sensing fairly small distances. Such devices are often called *proximity sensors* or *proximity detectors*.

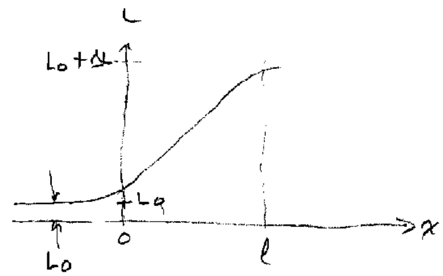


12.1.3 Solenoid Elements

Longer range, more linear inductive sensors can be made using a solenoid. With one end of the plunger inside the coil and the other outside, we effectively have two inductors in series: one with an iron core and the other with an air core.



The iron cored section will have a larger inductance per unit length due to the high permeability of iron compared to air. As the plunger is moved back in forth, the total inductance will change in an approximately linear fashion between the minimum value with the plunger removed and the maximum value with it fully inserted.

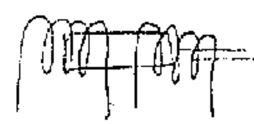


12.1.4 Differential Elements

As was the case with capacitive sensors, we can also build differential inductive sensors where one inductance decreases as the other increases. Again, this enables us to build simpler interface circuitry or improve measurement sensitivity.

12.1.4.1 Linear Differential Inductor

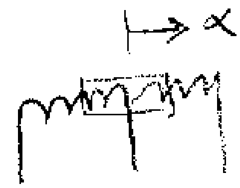
We can convert the solenoid inductive sensor to a differential inductor by using a center tapped coil and a plunger which is approximately half the total length of the coil.



We could use such a device in a simple voltage divider circuit. In this case we have

$$
\begin{aligned}
V &= \frac{Z_2}{Z_1 + Z_2} V_s \\
&= \frac{j\omega L2}{j\omega(L_1 + L_2)} V_s \\
&= \frac{L_2}{L_1 + L_2} V_s
\end{aligned}
$$

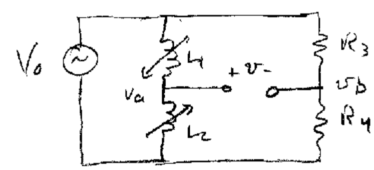
If L_0 is the inductance of each coil with the plunger removed, ΔL is the change in inductance when the plunger is fully inserted, and α is the position of the plunger as a fraction of the full excursion, then



$$\begin{aligned}
V &= \frac{L_0 + (\alpha + \frac{1}{2})\Delta L}{L_0 + (\alpha + \frac{1}{2})\Delta L + L_0 + (\frac{1}{2} - \alpha)\Delta L} V_s \\
&= \frac{V_s}{2L_0 + \Delta L}[(L_0 + \frac{1}{2}\Delta L) + \Delta L \alpha]
\end{aligned}$$

The change in V is linearly proportional to the change in α.

By using a bridge circuit, we can produce an output which is linear in α and which is zero when the plunger is centered.



$$\begin{aligned}
V &= V_a - V_b \\
&= \frac{j\omega L_2}{j\omega(L_1 + L_2)} V_0 - \frac{R_4}{R_3 + R_4} V_0 \\
&= \frac{L_2 R_3 + L_2 R_4 - L_1 R_4 - L_2 R_4}{(L_1 + L_2)(R_3 + R_4)} V_0 \\
&= \frac{L_2 R_3 - L_1 R_4}{(L_1 + L_2)(R_3 + R_4)} V_0
\end{aligned}$$

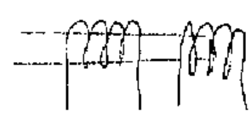
If we let $R_3 = R_4$

$$\begin{aligned}
V &= \frac{R_3}{(L_1 + L_2)(R_3 + R_4)} V_0 \\
&= K V_0 (L_0 + (\alpha + \frac{1}{2})\Delta L - L_0 - (\frac{1}{2} - \alpha)\Delta L) \\
&= K V_0 \Delta L \alpha
\end{aligned}$$

Note that in order to distinguish "positive" from "negative" values of V we must use a detector which is sensitive to the relative phase of V and V_0.

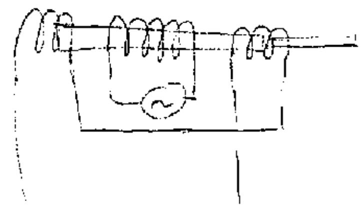
12.1.5 Transformer Elements

If we split the two halves of the coil into two separate windings and replace the plunger with a longer one, we get a *linear variable transformer*. If we connect an AC voltage source to the left hand winding, then the variation in mutual inductance will cause the amplitude of the voltage induced in the right hand winding to vary in proportion to the position of the plunger.



12.1.5.1 LVDT

By splitting the secondary winding into two halves, placing them on either side of the primary, and choosing a plunger of appropriate length, we produce a *linear variable differential transformer*, or *LVDT*. With the plunger centered, equal voltages are induced in each of the two secondaries, which cancel due to the polarity of their connection. The amplitude of the output will increase linearly with displacement on either side of this null, with the phase depending on the direction of displacement.



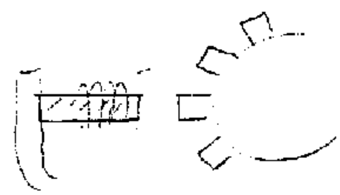
12.1.6 Magnetic Variable Reluctance

By combining a permanent magnet with a variable reluctance device, we can make a self generating sensor.

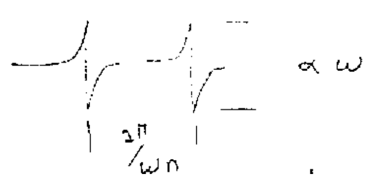
This structure is similar to the original Bell telephone tranceiver where the movable element was a thin diaphragm which moved in response to the impinging sound waves. The emf induced in the coil will be

$$v = -\frac{d\lambda}{dt} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\frac{\mathcal{F}}{\mathcal{R}} = \frac{N\mathcal{F}}{\mathcal{R}^2}\frac{d\mathcal{R}}{dt}$$

A variation on this idea can be used as a tachometer for measuring rotational velocity. In this device, the air gap, and hence the reluctance changes as the gear teeth pass by the stator pole. Again, a permanent magnet provides the source of mmf.

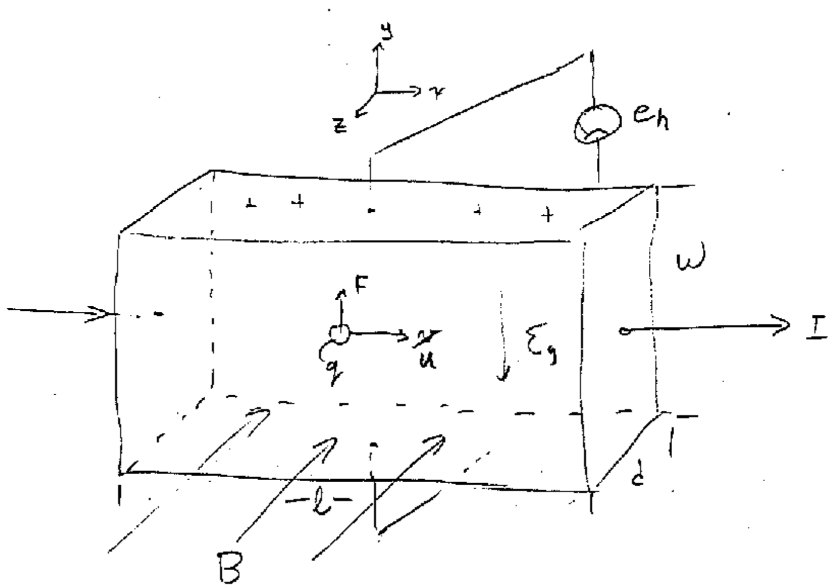


The resulting periodic variations in flux linkage produce a periodic output voltage whose frequency corresponds to the rate of rotation of the gear. Unfortunately, the amplitude of the signal also varies with speed, decreasing as the angular velocity decreases. For sufficiently small speeds, the output will be too small to be reliably measured.



12.2 The Hall Effect

In the figure below a rectangular block of conducting material has been placed in a magnetic field. A current I is flowing through the block parallel to the x axis.



The charge carrier q moving with velocity u in the field B will experience a force

$$\mathbf{F_1} = q(\mathbf{u} \times \mathbf{B})$$

This force will result in a vertical drift of the charge carriers, positive toward the top and negative toward the bottom of the block. As a result, charge will accumulate on the upper and lower surfaces, resulting in a field $\mathcal{E}_y$ which in turn will produce a force

$$\mathbf{F_2} = q\mathcal{E}_y$$

on the charge carriers. Since F_1 and F_2 must be balanced,

$$\mathcal{E}_y = uB$$

We can relate the charge velocity to the current

$$I = \frac{dQ}{dt} = \frac{qnwd\,dx}{dt} = qnwdu$$

where n is the carrier concentration in m^{-3}, so

$$\begin{aligned} u &= \frac{I}{qnwd} \\ \mathcal{E}_y &= \frac{IB}{qnwd} \end{aligned}$$

and the *Hall voltage*

$$e_h = \mathcal{E}_y w = \frac{IB}{qnd} = \frac{K}{d} IB$$

where $K = \frac{1}{qn}$ is the *Hall coefficient* and depends on the material.

In terms of the carrier mobility μ

$$u = \mu \mathcal{E}_x$$

so

$$e_h = \mathcal{E}_y w = uBw = \mu \mathcal{E}_x w B = \frac{w}{l} \mu e_x B$$

We can write this as

$$\frac{e_h}{e_x} = \frac{w}{l} \mu B$$

i.e. the ratio of the hall voltage e_h to the excitation voltage e_x is proportional to B.

In terms of resistivity ρ

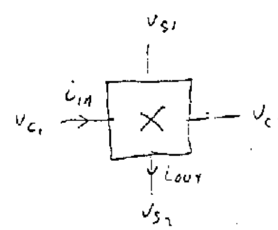
$$I = \frac{e_x}{R} = \frac{\mathcal{E}_x l}{\frac{\rho l}{dw}} = \frac{\mathcal{E}_x dw}{\rho}$$

so $\mathcal{E}_x = \frac{I\rho}{dw}$, and

$$\begin{aligned} u &= \mu \mathcal{E}_x \\ \mathcal{E}_y &= uB = \mu \mathcal{E}_x B = \frac{\mu\rho}{dw} IB \\ e_h &= \mathcal{E}_y w = \frac{\mu\rho}{d} IB \end{aligned}$$

i.e., $K = \mu\rho$, the Hall coefficient is the product of the mobility and the resistivity.

A *Hall device* is simply a piece of material with wires attached to the appropriate spots so that the Hall effect may be observed. A current (i_{in}) or voltage (v_c) is applied between one pair of parallel faces and the Hall voltage (v_s) is measured across the other pair.



The larger the Hall coefficient of the material, the more sensitive the Hall device. The Hall coefficient of copper is $K = 5 \times 10^{-11}$. If we let

$$\begin{aligned} B &= 2\text{T} \\ d &= 0.5\text{mm} \\ I &= 50\text{A} \end{aligned}$$

we get

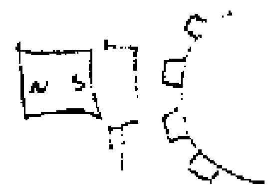
$$e_h = \frac{K}{d} IB = 10\mu\text{V}$$

Clearly, copper is not a vary good material for hall devices. Fortunately, there are a number of more suitable materials. Some values of K:

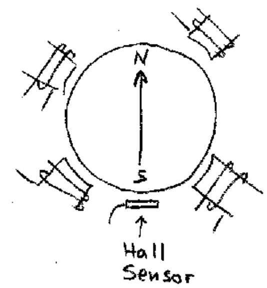
	K [m^3/A sec]	ρ [Ω m]	μ
Cu	5×10^{-11}	1.6×10^{-8}	
Bi	6×10^{-6}	10^{-6}	
Si	10^{-3}	10^{-2}	10^{-1}
Ge	4×10^{-3}	10^{-2}	
InSb	5.6×10^{-4}	3×10^{-5}	19

12.2.1 Hall Device Applications

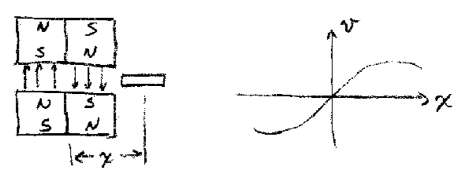
The response of the gear type rotational sensor can be extended down to DC (zero frequency) by replacing the coil with a Hall device. As before the passage of the gear teeth modulates the reluctance and hence the flux density, but now this flux is sensed directly rather than by its rate of change.



A brushless motor must monitor the rotor position in order to properly time the sequencing of currents in the stator windings. Since it is the magnetic pole of the rotor that forms the point of reference, a Hall sensor positioned near the rotor may be used to sense its position.



While a Hall device can be used with a single, isolated magnet as a proximity sensor, it can also be used as a continuous position sensor when combined with a magnetic circuit which produces a controlled variation of flux density with position. This configuration produces a strong field with an abrupt reversal of direction in the center. Because of the non-zero area of the Hall device, there will be a smooth transition from positive to negative output as the sensor is moved through the field.



This configuration achieves a similar result, but in this case, the field itself changes more slowly.