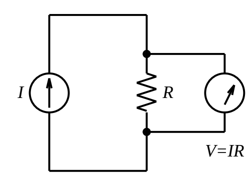
Chapter 13

Resistive Devices

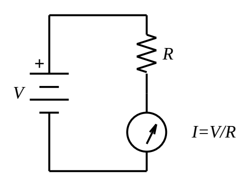
13.1 Measuring Resistance

The defining relationship for a resistor as a circuit element is Ohm's law: $V = IR$.

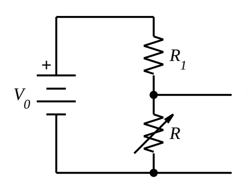
This suggests that we could measure a resistor by connecting it to a current source of known value and measuring the resulting voltage drop.



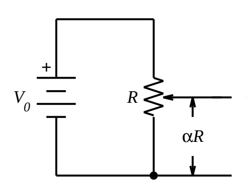
Or we could connect it to a known voltage source and measure the current.



Using only voltage sources and voltmeters, we can determine the value of an unknown resistance by comparing it to a known one in a voltage divider.



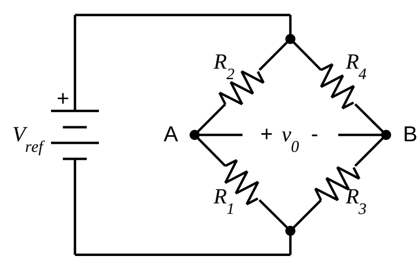
With a potentiometer, the total resistance remains constant and the division ratio, and hence the output voltage, varies linearly (for a linear potentiometer) with the slider position.



Other techniques for measuring resistance include combining with a known value of capacitance or inductance and determining the resulting time constant, or comparing to a known value of resistance. The standard circuit for comparing two resistances (or impedances) is the *bridge*.

13.1.1 Bridge Circuits

$$\begin{aligned}
v_a &= \frac{R_1}{R_1+R_2}V_{ref} \\
v_b &= \frac{R_3}{R_3+R_4}V_{ref} \\
v_0 &= v_a - v_b \\
&= \left[\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right]V_{ref} \quad (13.1) \\
&= \left[\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right]V_{ref} \quad (13.2)
\end{aligned}$$



For a balanced bridge, i.e. $v_0 = 0$ we get from (13.2):

$$R_1R_4 = R_2R_3$$

or

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

There are two techniques for using a bridge to measure the value of an unknown resistance:

Balance Measurement Measure the unknown R_1 by comparing it to a known R_2. This is done by adjusting $\frac{R_3}{R_4}$, for example with a linear potentiometer, until it balances $\frac{R_1}{R_2}$. Then $R_1 = \frac{R_3}{R_4}R_2$.

Deflection Measurement Start with a balanced or nearly balanced bridge. Determine changes in resistance values from the resulting changes in v_0.

13.1.1.1 Bridge Sensitivity

Start with the bridge in a balanced condition. If each of the resistors undergoes a small change in resistance δR_i, then the resulting change in output voltage will be

$$\delta v_0 = \sum \frac{\delta v_0}{\delta R_i}\delta R_i \tag{13.3}$$

From (13.1) we get

$$\begin{aligned}
\frac{\delta v_0}{\delta R_1} &= \frac{R_2}{(R_1+R_2)^2} V_{ref} \\
\frac{\delta v_0}{\delta R_2} &= -\frac{R_1}{(R_1+R_2)^2} V_{ref} \\
\frac{\delta v_0}{\delta R_3} &= -\frac{R_4}{(R_3+R_4)^2} V_{ref} \\
\frac{\delta v_0}{\delta R_4} &= \frac{R_3}{(R_3+R_4)^2} V_{ref}
\end{aligned}$$

Substituting these into (13.3) we get

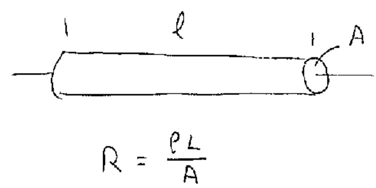
$$\frac{\delta v_0}{V_{ref}} = \frac{R_2\,\delta R_1 - R_1\,\delta R_2}{(R_1+R_2)^2} - \frac{R_4\,\delta R_3 - R_3\,\delta R_4}{(R_3+R_4)^2} \tag{13.4}$$

By exploiting this relationship, we can reduce temperature sensitivity or increase overall measurement sensitivity.

13.2 Mechanically Varying Resistance

The resistance of a conductor of length l and cross sectional area A is given by

$$R = \frac{\rho l}{A}$$



where ρ is the *resistivity* of the material. This is similar to the formulas for capacitance and inductance in that it depends on a length, an area, and a material property. This suggests that we can produce resistive sensors by finding ways to mechanically vary these three parameters.

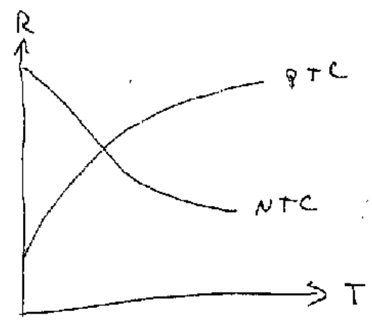
13.2.1 Varying Resistance by Changing Effective Resistivity

13.2.1.1 Temperature

Like most properties, the resistivity of a material varies with temperature. For a linear model of this variation, we get

$$R = R_0(1 + \alpha T)$$

where α is the temperature coefficient of resistivity. Most metals have a positive temperature coefficient (PTC), i.e. the resistance increases with increasing temperature. Semiconductors (including carbon) have a negative temperature coefficient (NTC); their resistance decreases with increasing temperature.



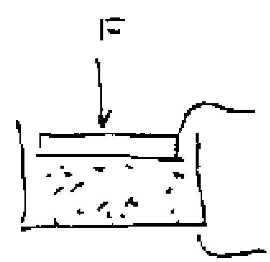
13.2.1.2 Light

Semiconductor materials exhibit significant changes in resistance when exposed to light. This will be considered later when we cover optical devices.

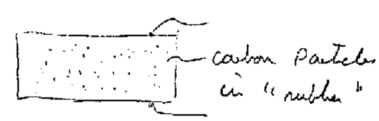
13.2.1.3 Pressure

When a material is stretched or compressed, its resistivity may change due to distortion of the crystal lattice. This phenomenon is called *piezoresistivity*, and is a significant factor in strain gages, to be discussed later.

It is also possible for the *effective* resistivity of a substance to change in response to pressure. In the carbon button microphone, a small pile or "button" of carbon granules is held between two metal plates. As the granules are squeezed together between the plates high localized pressure at the points of contact increases the contact area, thereby lowering the resistance.



The inconsistency and noise caused by the shifting of the carbon granules can be eliminated by imbedding them in an elastomer such as rubber, which holds them in a fixed relationship to each other, but still allows local deformation at the points of contact.

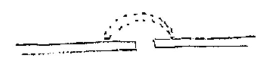


13.2.2 Varying Resistance by Changing Geometry

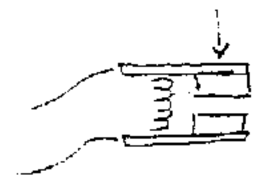
The other parameters effecting resistance are the length and cross sectional area. Devices which mechanically change one or the other or both of these will be discussed in subsequent sections.

13.3 Switches

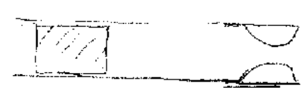
In a switch, the cross sectional area of a conducting path is abruptly reduced to zero when the switch is opened, and restored to its original value when the switch is closed.



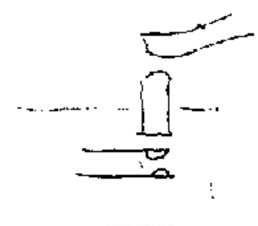
The contact points of a switch are usually given special treatment to maintain consistently low values of on-resistance in the face of wear, corrosion, and dirt. Sliver or gold plated contacts reduce electrical resistance and susceptibility to corrosion.



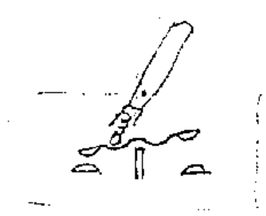
Switch contacts are usually shaped so as to give a small area of contact, thereby increasing the pressure for a given applied force. High pressure at the point of contact helps to displace dirt and surface films, increasing the reliability and lowering the resistance of the contact.



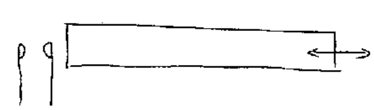
A switch may or may not return to its original electrical state when the mechanical stimulus is removed. A *momentary contact* switch remains closed only as long as mechanical force is applied. Examples include doorbell or elevator buttons and keyboard switches.



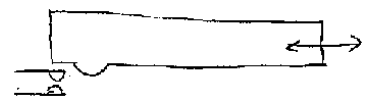
A toggle switch is a mechanical RS flip-flop: it will remain in its current state until mechanically switched. Once switched, the force required to switch states may be removed and the switch will remain in the new electrical state.



A switch used to sense the extreme position of a machine element is called a *limit switch.* If the element is electrically conductive, it may be used as one of the components of the switch, but usually a separate switch assembly is used.

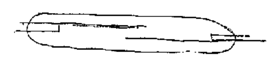


The end-on placement above can damage the switch if the motion being sensed is not mechanically limited. By actuating the switch with a bump or cam it is protected agains mechanical overrun.

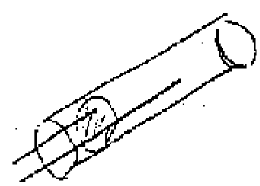


By using a rotary cam, it is also possible to sense rotational position.

A magnetic *reed switch* consists of two soft iron reeds sealed inside a glass cylinder. When placed in a magnetic field, reluctance forces will draw the two reeds together, closing the circuit. Because the contacts are sealed, reed switches are highly resistant to contact contamination from dirt and corrosion.

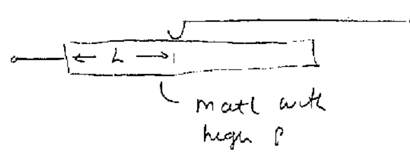


Another type of switch using contacts sealed in glass is the mercury tilt switch. As it is tilted either side of horizontal, the ball of mercury rolls from one end to the other, opening or closing the circuit.

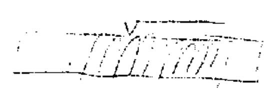


13.4 Potentiometers and Rheostats

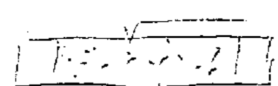
The effective length of a resistive element may be changed by sliding one of the contact points along the length of the material.



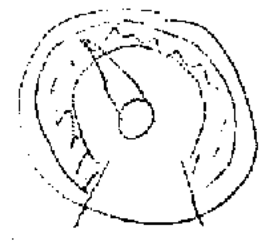
Because of the low resistivity of metals, variable resistors using metallic resistance elements are usually made by wrapping many turns of fine wire around an insulating mandrel.



The higher resistivity of carbon allows larger values of resistance in a compact aspect ratio. Carbon resistance elements typically consist of carbon particles in an organic binder applied to an insulating substrate.



A *potentiometer* is a variable resistor where fixed contacts are made to both ends of the resistive element, in addition to the sliding contact. This makes it particularly suitable for use in voltage divider circuits. Potentiometers may be either linear or rotational devices.



13.5 Strain Gages

In a strain gage, the resistive element is mechanically deformed, resulting in simultaneous changes in length, cross-sectional area, and resistivity.

The resistance of a round wire of length l and diameter D is

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi \frac{D^2}{4}}$$

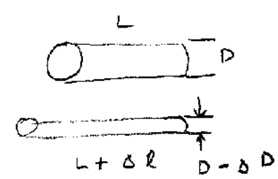
Taking logarithms, we get

$$\log R = \log \rho + \log l - 2 \log D - \log \frac{\pi}{4}$$

Differentiating

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - 2\frac{dD}{D} \tag{13.5}$$

The change in length per unit length is called *strain* and is given the symbol ϵ. Strain in the direction of deformation ($\frac{dl}{l}$ in this case) is called *axial* strain, while strain perpendicular to this direction ($\frac{dD}{D}$ here) is called *transverse*. If the applied force tends to stretch the wire, it produces *tensile* strain; the accompanying transverse strain will be *compressive*. Axial and transverse strain are related by a property of the material called *Poisson's ratio*:



$$\nu = -\frac{dD/D}{Dl/l}$$

From (13.5)

$$\begin{aligned}\frac{dR}{R} &= \frac{d\rho}{\rho} + \frac{dl}{l} + 2\nu\frac{dl}{l} \\ &= \frac{d\rho}{\rho} + (1 + 2\nu)\frac{dl}{l} \\ &= K\frac{dl}{l} = K\epsilon\end{aligned}$$

where

$$K = \frac{dR/R}{dl/l} = \frac{d\rho/\rho}{dl/l} + (1 + 2\nu)$$

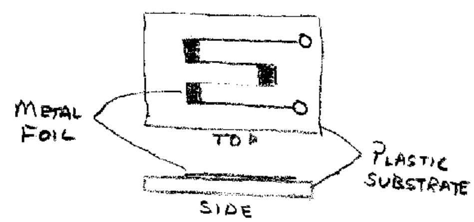
is called the *gage factor*.

For most metals, $\nu \approx 0.3$ so that $K = 1.6 + \frac{d\rho/\rho}{dl/l}$. Since the observed value of K in metals is typically about 2 and can be as high as 6, piezoresistivity, represented by $\frac{d\rho/\rho}{dl/l}$ is an important component of strain gage behavior. For semiconductors, K ranges between 50 and 200.

13.5.1 Strain Gage Construction

A *strain gage* is a piece of resistive material which is physically bonded to the object whose strain is to be measured. It has a long length and small cross-sectional area to increase electrical resistance and reduce its mechanical influence on the object being measured.

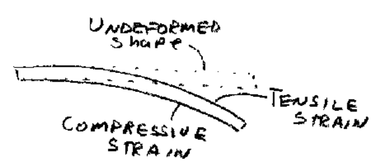
Strain gages were originally made of fine wire, wrapped in a serpentine fashion around pins attached to the object being measured. Modern strain gages consist of a thin metal film on an insulating substrate. The gage is adhesively bonded to the surface of the object being measured, coupling its strain into the gage.



13.5.2 Strain Gage Applications

13.5.2.1 Measuring Strain and Stress

When a bar is bent, the outer surface experiences tensile strain and the inner surface compressive strain, with the amount of strain proportional to the amount of bending. This is used in some VR gloves (such as the Nintendo Power Glove) to sense the motion of the wearer's fingers.

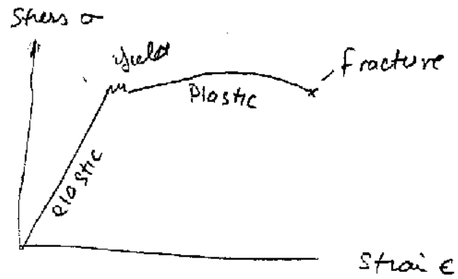


When stress is applied to a material it will deform, i.e. undergo strain. For sufficiently small values of stress, the relationship between stress and strain is linear and is governed by *Hooke's law*:

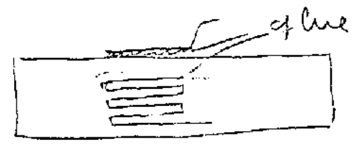
$$\sigma = E\epsilon$$

where σ is the *stress* in N/m^2 and E is *Young's modulus*. The values of stress and strain for which this relation holds is called the *elastic region*. Elastic deformation is reversible; if the stress is removed, the material will return to its original length.

In brittle materials, the elastic region ends abruptly when the material fails by fracturing. In ductile materials such as metals, elastic behavior continues until the material reaches the *yield point* where the material begins to deform *plastically*. Plastic deformation is irreversible; once plastically deformed, an object will not return to its original size. Plastic deformation continues until the material reaches its *ultimate tensile strength* whereupon it fractures.

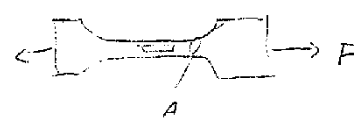


In order to insure a safe design, it is necessary that the stresses in a part remain below the yield point by an adequate margin. Since it is difficult to directly measure localized stress in a complex part, strain is measured instead by applying strain gates to stress concentration points.

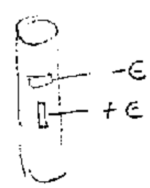


13.5.3 Measuring Force

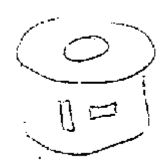
If we apply longitudinal force to a bar of cross-sectional area A then the resulting stress will be $\sigma = \frac{F}{A}$. If this is within the elastic region for the material, then by Hooke's law the resulting strain will be $\epsilon = \frac{\sigma}{E}$. By measuring this with a strain gage, we can determine the applied force. A device for determining force by measuring the resulting strain is called a *load cell*



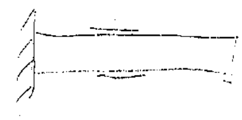
A load cell consisting of a thin rod in tension is called, simply enough, a *tension rod*. A strain gage placed with its sensitive axis along the length of the rod will measure transverse strain, while one placed circumferentially will measure compressive strain.



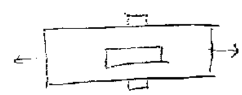
Applying excessive compressive force to a long, thin rod may cause it to buckle, so load cells for compressive force often take the form of a *compression column*.

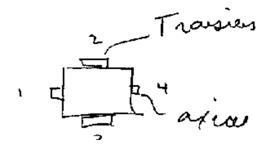


Another form of load cell is the bending beam. In this case the force is applied transversely to the unsupported end of a cantilevered beam and the strain of the resulting bending is measured. By placing one gage on the top and another on the bottom of the beam, both the tensile and the equal and opposite compressive strain may be utilized (e.g. in a half-bridge).



We can place additional gages to create a full bridge containing four sensing elements. With the gages placed as shown on a tension rod, and if





$$\delta R_1 = \delta R$$

then

$$\begin{aligned} \delta R_2 &= -\nu \delta R \\ \delta R_3 &= -\nu \delta R \\ \delta R_4 &= \delta R \end{aligned}$$

Then from (13.4)

$$\frac{\delta v_0}{V_{ref}} = 2(1+\nu)\frac{\delta R}{4R}$$

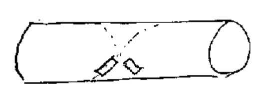
This is $k = 2(1+\nu)$ times more sensitive than the 1/4 bridge configuration using a single gage. The factor k is called the bridge constant. The overall sensitivity is

$$\frac{\delta v_0}{V_{ref}} = C\epsilon$$

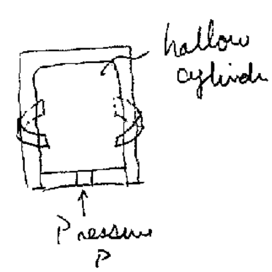
where $C = \frac{k}{4}K$.

13.5.4 Additional Strain Gage Sensors

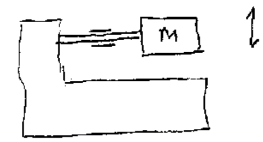
Torque. By placing pairs of strain gages at 45° to the axis of a shaft we can measure the torque being transmitted by sensing the torsional stress.



Pressure. A hollow cylinder will bulge outward if the pressure inside exceeds that outside. Strain gages placed around the "waist" if the cylinder can measure the resulting hoop stress and hence determine the pressure.



Acceleration. By attaching a known mass to the end of a bending beam load cell, we can measure the inertial force produced by changes in velocity of the base of the beam, producing an *accelerometer*



Multi-axis Force/Torque. With the rim of a four-spoked wheel fixed, a force or torque applied along or about any of the coordinate system axes will result in different combinations of strain in the 16 gages shown in the figure. By forming appropriate linear combinations of these gage readings, it is possible to simultaneously determine all six of these force and torque values.