Chapter 5

Electrostatic Devices

5.1 Electrostatics

If the magnetic field is zero, and the charge q is fixed, 4.1 reduces to

$$\mathbf{F} = q\mathbf{E} \tag{5.1}$$

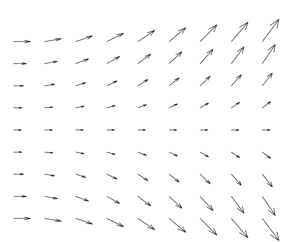
and Maxwell's equations become

$$\nabla \times \mathbf{E} = 0 \quad \text{or} \quad \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

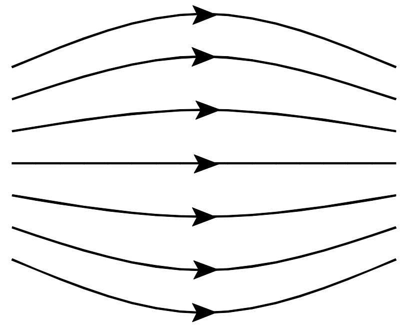
$$\nabla \cdot \mathbf{D} = \rho \quad \text{or} \quad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

The electric field $\mathbf{E}$ is a vector field: at any point in space it has both a magnitude and a direction.

One way to visualize this field would be to use *velocity vectors*: at every point in space (or more conveniently on a regular grid of points) we draw an arrow whose direction is the same as the direction of the field and whose length is proportional to the magnitude of the field.



However, field diagrams are traditionally drawing using *field lines*. This is a set of continuous, directed lines whose tangent at any point lies in the direction of the field at that point. Electric field lines begin on positive charge and terminate on negative charge. Although the magnitude of the field is not directly represented in such a drawing, we will see that the *density* of the field lines is proportional to the magnitude of the field.



5.1.1 Coulomb's Law

The force between two charged particles separated by a distance r is directed along the line between them with magnitude given by *Coulomb's Law*:

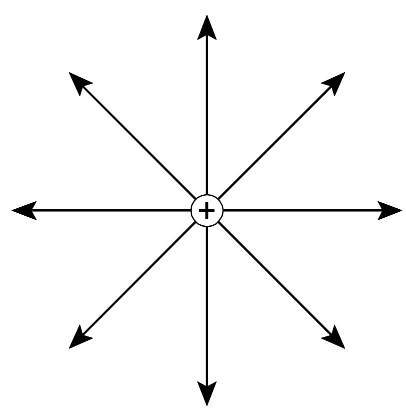
$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2} \tag{5.2}$$

where Q_1 and Q_2 are the charges in coulombs and ϵ_0 is a constant of proportionality known as the *permittivity* of free space. Charge is a signed quantity, and the force is repulsive (positive) or attractive (negative) depending on whether the two charges have the same or opposite signs.

Since the field describes the force on a *test charge* due to the presence of the other charges in the system, Coulomb's law gives us the field due to a single point charge of charge Q_2.

$$\mathbf{E_2} = \frac{\mathbf{F}}{Q_1} = \frac{Q_2}{4\pi\epsilon_0 r^2}\mathbf{a_r} \tag{5.3}$$

where $\mathbf{a_r}$ is the unit vector directed from the location of Q_1 to Q_2.



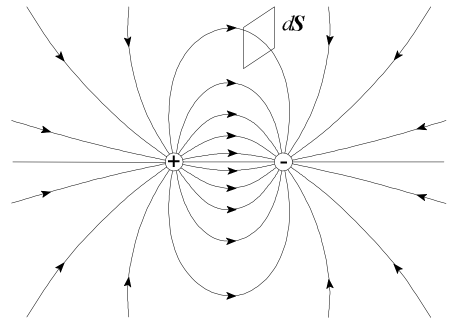
In general, the charge in a system will be in the form of a continuous charge density ρ rather than discrete point charges. In this case, the field at a point is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\, dv}{r^2}\mathbf{a_r}(v) \tag{5.4}$$

where $\mathbf{a_r}(v)$ is the unit vector in the direction of volume element dv. Although this allows us to find the field for any arbitrary charge distribution, for many cases a much quicker answer can be gotten by using Gauss' law and the notion of electric flux.

5.1.2 Electric Flux

Historically, the field lines were intended to represent the flow of some electrical essence and hence were called *lines of flux*. Let's consider each unit or *line* of flux to originate on a unit of positive charge, terminate on a unit of negative charge, and to be everywhere parallel to the direction of the **E** field. In this case, the units of flux are coulombs.



If we have an area $d\mathbf{S}$ then we say that the flux $d\Psi$ passing through $d\mathbf{S}$ is equal to the number of field lines passing through $d\mathbf{S}$.

Although the flux itself is a scalar quantity, its associated lines have direction. We embody this in the notion of *electric flux density*: $\mathbf{D} = \frac{d\Psi}{dS}\mathbf{a}_n$ or $d\Psi = \mathbf{D} \cdot d\mathbf{S}$.

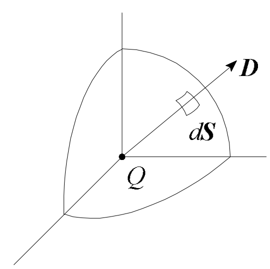
If we have a closed surface S surrounding a total charge of Q then the total flux through this surface must be equal to Q: $\Psi_S = Q_{enclosed}$. But $\oint_S \mathbf{D} \cdot d\mathbf{S} = \oint_S d\Psi = \Psi = Q$. If ρ is the charge density, then $Q = \int_V \rho dv$. The resulting relationship

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv \tag{5.5}$$

is one form of *Gauss' Law*.

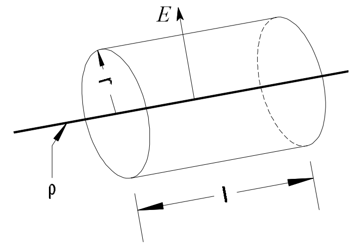
5.1.3 Using Gauss' Law to find E

For a point charge Q at the center of a sphere of radius r we know by symmetry that $\mathbf{D}$ is uniform and normal to the surface S, so $Q = \oint \mathbf{D} \cdot d\mathbf{S} = D \oint dS = D4\pi r^2$ or $D = \frac{Q}{4\pi r^2}$ or $\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$. But remember that $\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r$ so that in free space $\mathbf{E} = \dfrac{\mathbf{D}}{\epsilon_0}$.



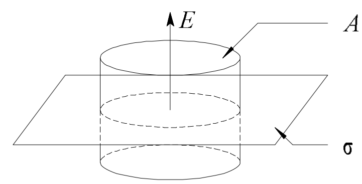
In situations with a high degree of symmetry, we can use Gauss' law to find **E** without having to evaluate 5.4.

Infinite line. To find the field of an infinite line of charge (of density ρ C/m) place a cylindrical gaussian surface of radius r around a length l of the line. The enclosed charge is $Q_{encl} = \rho l$. By symmetry, the field must be directed radially outward and independent of angle. Since no flux passes through the ends of the cylinder $A = 2\pi rl$, and since it is uniform $D = \frac{Q}{A} = \frac{\rho l}{2\pi rl}$. So we have $\mathbf{D} = \frac{\rho}{2\pi r}\mathbf{a_r}$ and



$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon_0} = \frac{\rho}{2\pi\epsilon_0 r}\mathbf{a_r} \tag{5.6}$$

Infinite plane. To find the field of an infinite sheet of charge (of density σC/m^2) use a "gaussian pillbox" with a top and bottom surface area of A. By symmetry, all the flux emerges perpendicular to the horizontal faces, so $\Psi = 2AD = A\sigma$, $D = \frac{\sigma}{2}$ and



$$\mathbf{E} = \frac{\sigma}{2\epsilon_0}\mathbf{a_n} \tag{5.7}$$

5.1.4 The Electric Field of Simple Charge Configurations

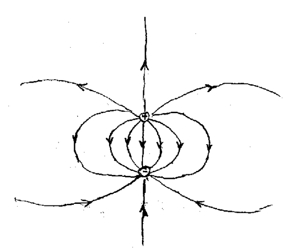
By combining Gauss' law with superposition we can get the field of several useful charge configurations as shown in cross section in Figure 5.1.

5.1.4.1 Parallel Sheets of Charge

A particularly useful configuration is that shown in Figure 5.1f, the field between two parallel sheets of charge. Here $E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$ between the sheets and $E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$ outside. The field is uniform, parallel, and directed perpendicular to the sheets.

5.1.4.2 The Dipole

A charge configuration that looks very similar (at least in cross section) to Figure 5.1c is that of two charges of equal magnitude q and opposite sign separated by a distance d.



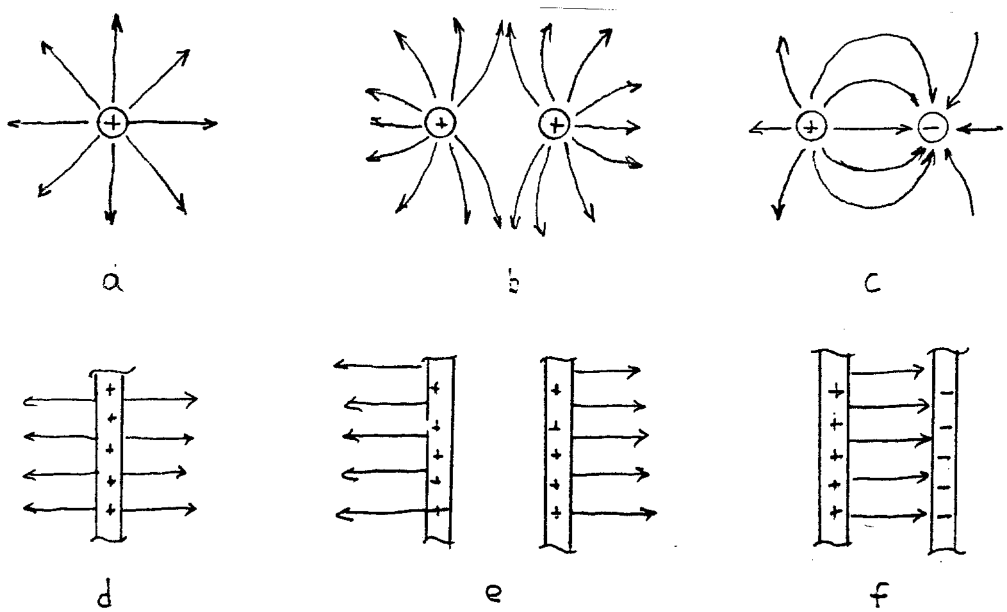
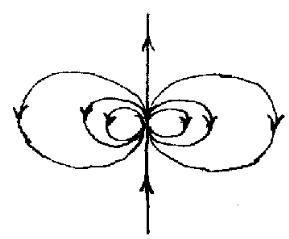


Figure 5.1: Some simple electric field configurations. (a) an infinite line. (b) and (c) parallel lines. (d) an infinite plane. (e) and (f) parallel planes.

As we move further away, or move the charges closer together we get the limiting case of the *electric dipole*. The product $p = qd$ is called the *dipole moment*.



5.1.5 Conductors

A *conductor* is a material, usually a metal, which has a large number of *free electrons*. These electrons are not totally free: although they may move freely inside the material, they are constrained to remain within its boundaries.

This charge mobility gives conductors several interesting properties for the case of a static charge distribution:

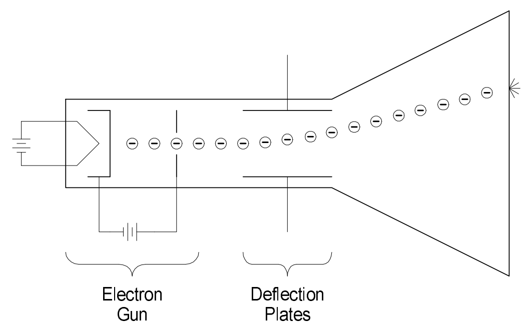
1. Excess charge must reside on the surface.

2. The electric field in the interior is zero.

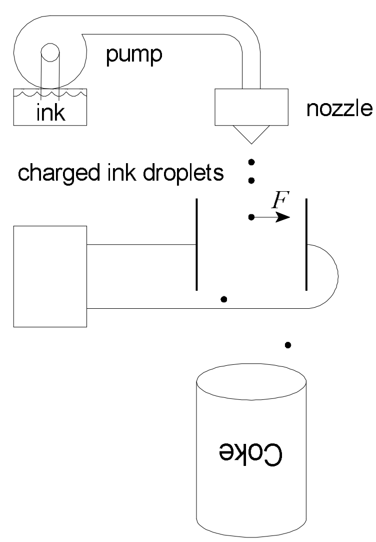
3. The tangential component of the field at the surface is zero.

4. Field lines are perpendicular to the surface.

5.1.6 Some E Field Devices



CRT. One electrostatic "device" which relies directly on the force on a charged particle produced by an electric field is the electrostatic deflection *cathode ray tube* (CRT). The cathode is heated to the point where its free electrons have sufficient thermal energy to escape from the surface, forming an "electron cloud" around it. These electrons are accelerated by the field produced by the *electron gun* and formed into a thin beam directed toward the face of the tube. This is coated with a material called a *phosphor* which emits light when struck by sufficiently energetic electrons. With no other fields present, this beam will strike the center of the face of the tube, forming a bright spot there. Two sets of parallel plates, one horizontal and the other vertical, are placed around the beam near the cathode. If positive charge is placed on the upper horizontal plate and a corresponding negative charge on the bottom plate, a vertically oriented field will be produced between them. As the beam of electrons pass through this region, they will experience an upward force (since their charge is negative) and hence an upward acceleration. This will *deflect* the beam in a vertical direction and move the spot upward on the face of the tube. Similarly, the two vertical plates can deflect the beam horizontally. By combining these two motions, we can trace out graphs of electrical signals as in an oscilloscope, or raster images as in a television display.



Ink jet printer. The small ink jet printers used at home or in the office have a a separate nozzle for each row of dots, with sometimes over a hundred individual nozzles on a print head. Drops of ink are produced on demand and ejected in a straight line and travel only a small distance to the paper. Many of the large industrial printers used for labeling such things as the date code on the bottoms of soft drink cans produce a continuous stream of ink droplets which are given an electric charge as they leave the nozzle. These are directed between a pair of deflection plates similar to those in the CRT which provide vertical positioning for the drops (horizontal positioning is controlled by the motion of the product on the convayer belt).

5.1.7 Electric Potential

If we move a test charge around in the field, work must be done on it. In moving charge Q_T from point a to point b in field E, the work done on Q_T is

$$\begin{aligned} W &= -\int_a^b \mathbf{F} \cdot \mathbf{dl} \\ &= -\int_a^b \mathbf{E} Q_T \cdot \mathbf{dl} \end{aligned} \tag{5.8}$$

where the minus sign indicates that the applied force is the negative of the force generated by the field. So the work per unit charge is

$$\frac{W}{Q_T} = -\int_a^b \mathbf{E} \cdot \mathbf{dl}$$

The electrostatic force is *conservative* so the work is independent of the path taken from a to b. Hence we can define a scalar potential function φ such that

$$\frac{W}{Q_T} = -\int_a^b \mathbf{E} \cdot \mathbf{dl} = \varphi(b) - \varphi(a)$$

This implies that

$$\mathbf{E} = -\nabla\varphi$$

i.e. the electric field is the gradient of the potential. We refer to $\varphi(b) - \varphi(a)$ as the *potential* of b with respect to a. The unit of electrical potential is the *volt*.

Energy Storage in an Electric Field If we attempt to construct a configuration of charge, it will be necessary to perform work on the component charges as they are moved through the field generated by those previously assembled. We may consider the energy expended in this construction as being *stored* in the resulting field.

5.2 The Capacitor

The most common device for storing electrostatic energy is the *capacitor*, shown schematically in Figure 5.2.

If we assume that d is small enough with respect to the size of the plates that the curved field lines at the edges (called the *fringing field*) may be neglected, we can derive the field from the superposition of two sheets of charge as:

$$E = \frac{\sigma}{\epsilon_0}$$

between the plates and zero elsewhere. Since

Figure 5.2: A parallel plate capacitor

$$E = -\nabla\varphi = -\frac{\partial\varphi}{\partial x} = \frac{\Delta\varphi}{d}$$

the potential difference between the plates is

$$V = \Delta\varphi = Ed = \frac{\sigma d}{\epsilon_0} = \frac{d}{\epsilon_0 A}Q$$

Where Q is the total charge on the plates. Since $d/\epsilon_0 A$ depends only on the geometry of the device, we can define the parameter

$$C = \frac{\epsilon_0 A}{d} \tag{5.9}$$

as its *capacitance*, so that

$$V = \frac{Q}{C} \text{ or } Q = CV \tag{5.10}$$

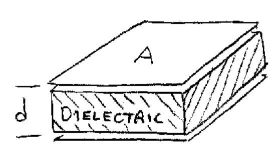
The unit of capacitance is the *farad*.

Energy Storage in a Capacitor The voltage V on a capacitor represents the work per unit charge required to charge it. So the total work done in charging it is the potential energy stored:

$$W = \int V\, dQ = \int \frac{Q}{C}\, dQ = \frac{1}{2C}Q^2 = \frac{1}{2}CV^2 \tag{5.11}$$

5.2.1 Dielectrics

If we fill the space between the plates of Figure 5.2 with a substance such as glass, we find that it takes more charge to create the same field (or that the same potential difference accumulates more charge), i.e. the capacitance has increased. Such substances are called *dielectrics* and the ratio by which the capacitance is increased is called the *dielectric constant* of the material. In other words



$$C = \epsilon_r \frac{\epsilon_0 A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon A}{d} \tag{5.12}$$

where ϵ is the *permittivity*, and ϵ_r is the dielectric constant or *relative permittivity* of the material.

Table 5.1 gives a few examples of dielectric constants and Section A.4.2 has a more comprehensive list.

Material	ϵ_r
air	1.00054
glass	5.3 - 7.5
water	80
$BaTiO_3$	1000 - 10000

Table 5.1: Some dielectric constants

.

5.3 Capacitive Sensors

In addition to being a device for storing energy electrically and for producing a uniform parallel electric field, a capacitor is also a circuit element with the following defining equations:

$$
\begin{aligned}
Q &= CV \\
i &= C\frac{dv}{dt} \\
Z_C &= \frac{1}{j\omega C} = jX_C
\end{aligned}
$$

Mechanically induced changes in the capacitance will be reflected in electrical changes in the circuit, providing a sensing mechanism.

5.3.1 Measuring Capacitance

There are several ways we can determine the value of an unknown capacitor in a circuit.

1. Compare to a known capacitor.
2. Charge with a known current and measure $\frac{dv}{dt}$.
3. Measure the time constant when combined with a known resistor.
4. Measure the resonant frequency when combined with a known inductor.

5.3.2 Mechanically Varying Capacitance

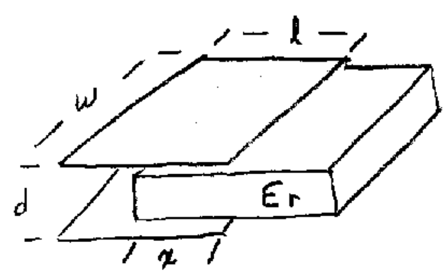
In Equation 5.12 there are three variables which effect the value of the capacitance and each of these can be coupled to a mechanical variable which we wish to measure.

5.3.2.1 Varying the Dielectric Constant

Like most other properties, the dielectric constant of a material varies with temperature, so in theory we could use a capacitor as a temperature sensor. However, there are much better ways to measure temperature, and one could argue that temperature isn't a mechanical variable anyway, so we won't try to do this. Instead, let's see how we can mechanically vary the *effective* dielectric constant of the space between the plates.

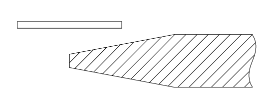
If we slide a slab of material with dielectric constant ϵ_r back and forth between the plates then the total capacitance is given by

$$\begin{aligned} C &= \frac{\epsilon_0 w(l-x)}{d} + \frac{\epsilon_r \epsilon_0 w x}{d} \\ &= \frac{\epsilon_0 w}{d}(l - x + \epsilon_r x) \\ &= \frac{\epsilon_0 w}{d}(l + x(\epsilon_r - 1)) \\ &= \frac{\epsilon_0 w}{d} l + \frac{\epsilon_0(\epsilon_r - 1)w}{d} x \\ &= C_0 + \frac{\epsilon_0(\epsilon_r - 1)w}{d} x \end{aligned}$$

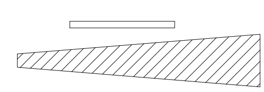


where C_0 is the value of the capacitance with no material between the plates and $0 < x < w$.

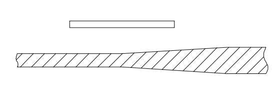
There are a number of variations on this idea. We can change the shape of the C vs. x curve by using a dielectric with non uniform thickness.



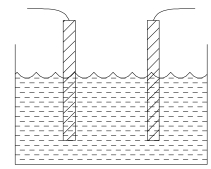
We can increase the range of measurement by reducing the slope of the taper.



As the taper goes to zero, we can use this structure to measure the *thickness* of the material between the plates.



We can also use a liquid dielectric material in which case we get a liquid level sensor.

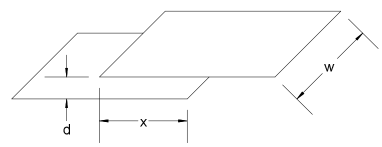


The high dielectric constant of water provides increased sensitivity suggests several novel applications. For example it could be used to monitor the moisture content of paper in a paper mill.

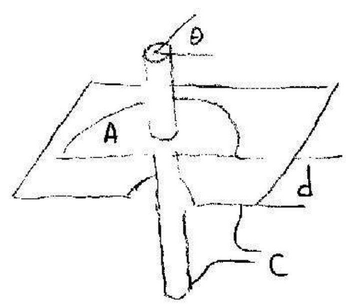
5.3.2.2 Varying the Area of the Plates

If instead of sliding a movable dielectric between fixed plates we move one of the plates with respect to the other, we get the following linear relationship between displacement and capacitance:

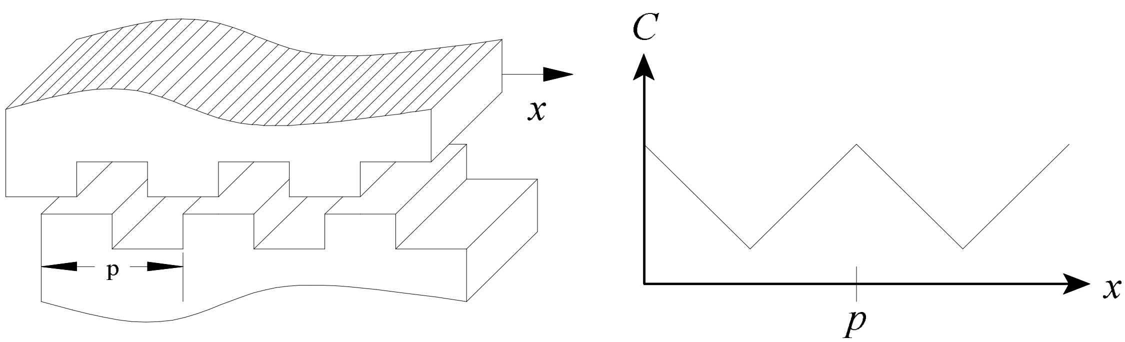
$$C = \frac{\epsilon_0 w}{d} x$$



We can also use this approach to sense angular position. Here $C = \frac{\epsilon_0 r}{d}\theta$



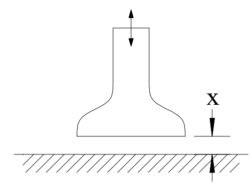
A particularly useful concept is the notion of serrated electrodes.



Here we can make the *range* as large as we like by extending the total length of the plates, while making the *sensitivity* as high as we like by decreasing the pitch of the serrations. In exchange for this we must increase the complexity of the interface circuitry to keep track of which cycle we're in. This concept will recur in a number of devices, particularly stepper motors and incremental optical encoders.

5.3.2.3 Varying the Separation of the Plates

We can capacitively sense the separation between two flat conductive surfaces which are insulated from each other: $C = \frac{\epsilon_0 A}{x}$ where A is the area of the smaller of the two surfaces. This has the disadvantage that the capacitance is nonlinear in x but provides a sensitive way of measuring very small displacements.



We can either accept this nonlinearity or restrict ourselves to displacements which are small compared to the total separation, in which case the relation is approximately linear. Also note that although the capacitance itself is nonlinear in x, the capacitive reactance ($X_C = \frac{1}{\omega C}$) is linear in x.

By making the movable plate a flexible diaphragm, forming one wall of a sealed chamber, we can use this approach to measure pressure.

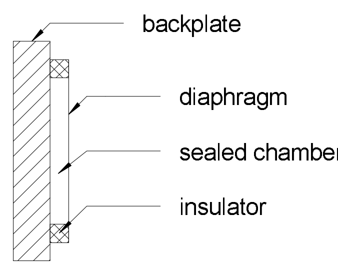
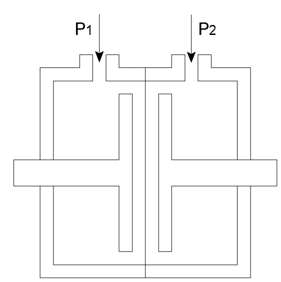
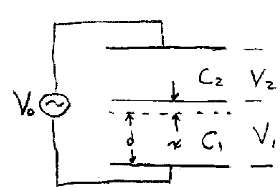


Figure 5.3:

By placing the diaphragm between two separate volumes we can measure differential pressure. If in addition we place the diaphragm between two fixed plates, we create a *differential capacitor.*



A differential capacitor can be made to yield a linear relationship between displacement and output voltage with the following circuit. The two outer plates are connected to a sinusoidal AC voltage source $v(t) = V_0 \cos(\omega t)$. Let C_1 and C_2 be the capacitances between the movable plate and the first and second fixed plates respectively. Then $C_1 = \frac{\epsilon_0 A}{d+x}$ and $C_2 = \frac{\epsilon_0 A}{d-x}$. These two



capacitances form a voltage divider with

$$V_1 = \frac{X_{C_1}}{X_{C_1} + X_{C_2}} V_0 = \frac{1/\omega C_1}{1/\omega(C_1 + C_2)} V_0 = \frac{C_1 + C_2}{C_1} V_0 = \frac{\epsilon_0 A/2d}{\epsilon_0 A/(d+x)} V_0 = \frac{d+x}{2d} V_0$$

and

$$V_2 = \frac{X_{C_2}}{X_{C_1} + X_{C_2}} V_0 = \frac{1/\omega C_2}{1/\omega(C_1 + C_2)} V_0 = \frac{C_1 + C_2}{C_2} V_0 = \frac{\epsilon_0 A/2d}{\epsilon_0 A/(d-x)} V_0 = \frac{d-x}{2d} V_0$$

If we take as the output the difference $V = V_1 - V_2$ then

$$V = \frac{d+x}{2d} V_0 - \frac{d-x}{2d} V_0 = \frac{x}{d} V_0$$

which is linear in x.

5.4 Forces in an Electrostatic System

If we attach one of these capacitive sensors to a mechanism, we find that rather than being completely passive, it exerts a small force. This should not be surprising, since the charges on the plates produced by the applied voltage are of opposite sign and therefore will attract.

Consider the system in Figure 5.4 consisting of two parallel conducting plates having an equal but opposite charge of Q coulombs. There are several approaches we could take to determine the forces between the plates:

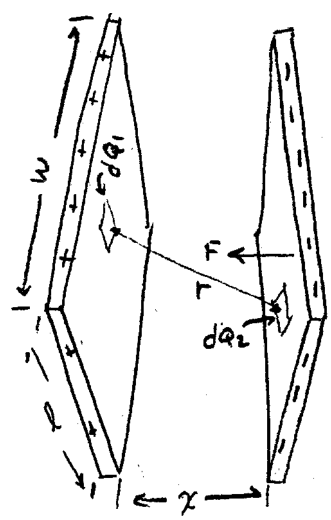


Figure 5.4: A simple electrostatic device.

Coulomb's Law For each element of charge dQ_2 on the right hand plate, we could compute the force exerted by each element of charge dQ_1 on the left hand plate. Summing over all charge on plate 1 gives the total force on dQ_2, and summing over all charge on plate 2 gives the total force:

$$F = \frac{1}{4\pi\epsilon_0} \int\int \frac{dQ_1 dQ_2}{r^2} \tag{5.13}$$

Although this is elegant in appearance, the prospect of actually having to evaluate it is not an appealing one.

Finding Force from the Electric Field One reason we developed the concept of the electric field was to avoid having to perform such integrations. If we assume w and l are large compared to x, then we may approximate the field of the left hand plate as that of an infinite sheet, in which case, from (5.7)

$$E_1 = \frac{\sigma}{2\epsilon_0} = \frac{Q/wl}{2\epsilon_0} = \frac{Q}{2wl\epsilon_0}$$

Since the total charge on the right hand plate is $-Q$, we have from (5.1)

$$F = -QE_1 = -\frac{Q^2}{2wl\epsilon_0} = -\frac{Q^2}{2A\epsilon_0} \tag{5.14}$$

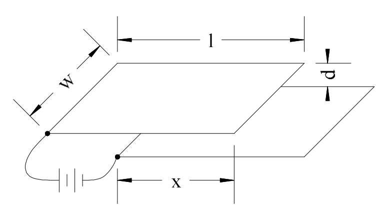
where A is the area of the plates. Note that the force produced is independent of the separation of the plates.

Equation 5.14 gives the force in terms of the charge on the plates, but normally we would drive them from a voltage source. Since $Q = CV$ and Substituting into 5.14

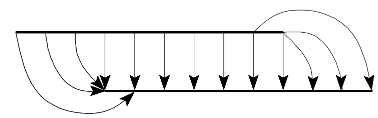
$$F = -\frac{Q^2}{2A\epsilon_0} = -\frac{{\epsilon_0}^2 A^2 v^2}{2d^2 A\epsilon_0} = -\frac{\epsilon_0 A}{2}\frac{v^2}{d^2} \tag{5.15}$$

Notice that in this case the force *does* depend on the distance between the plates. This does not contradict our previous results since they were determined assuming constant charge, and in order to maintain constant charge as the plates are moved, it would be necessary to vary the applied voltage.

We have been assuming that the two plates are constrained so that they can only move closer or further apart, not from side to side. Suppose instead that we constrain them so that their separation is fixed but that they are free to move in the x direction.



The resulting field might look something like this. If we knew the charge distribution, we could use 5.13 to find the force, but since the charges in the conducting plates are mobile, the charge density is no longer uniform. Qualitatively, we can think of the force in an electric field acting in such a way as to shorten the field lines. In this case, it is clear that in addition to the force tending to move the plates together vertically (which is resisted by the constraints) there is a net force tending to slide them into alignment horizontally.



It is possible to quantify this notion of *tensile stress*, but we would still have to find the field in order to apply it. A much simpler approach, which gives accurate results when the horizontal separation is large compared to the vertical, uses a family of techniques called *energy methods*.

Finding Force Using Conservation of Energy If we consider an electromechanical device as a system which accepts electrical energy as input and produces mechanical energy as an output, we may write the following *energy balance* relation based on conservation of energy:

$$\begin{matrix} Energy \\ input \end{matrix} = \begin{matrix} Energy \\ lost \end{matrix} + \begin{matrix} Increase\ in \\ stored\ energy \end{matrix} + \begin{matrix} Energy \\ output \end{matrix}$$

For an infinitesimal change in mechanical displacement, dx, the corresponding changes in energies are related by

$$dW_e = dW_l + dW_f + dW_m \tag{5.16}$$

where W_e, W_l, W_f, and W_m are the electrical input energy, energy lost (as heat), energy stored in the field, and mechanical output energy, respectively.

Since $dW_m = Fdx$ we may determine the force by

$$F = \frac{dW_m}{dx} \tag{5.17}$$

if we can evaluate the input, lost, and stored energy terms in (5.16).

In analyzing the mechanism of force production, we may consider the electrical losses to take place in the electrical circuitry before the electrical input, and the mechanical losses to take place in the mechanism being driven. If we consider the remaining lossless system, we have

$$dW_e = dW_f + dW_m \tag{5.18}$$

For the case of the isolated plates, $dQ = 0$, i.e. no electrical energy is added. So

$$dW_m = -dW_f$$

and

$$F = -\frac{\partial W_f}{\partial x}$$

From (5.11)

$$W_f = \frac{Q^2}{2C}$$

so

$$F = \frac{Q^2}{2C^2}\frac{\partial C}{\partial x}$$

But from (5.9)

$$C = \frac{\epsilon_0 A}{x}$$

Putting it all together:

$$\begin{aligned} F &= \frac{Q^2}{2\left(\frac{\epsilon_0 A}{x}\right)^2}\frac{\partial}{\partial x}\left(\frac{\epsilon_0 A}{x}\right) \\ &= -\frac{Q^2}{2\epsilon_0 A} \end{aligned} \tag{5.19}$$

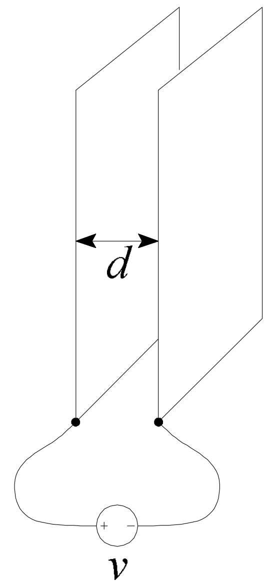
The case where the plates are connected to a source proceeds in a similar fashion, but dW_e is no longer zero. Details are left for a homework problem, along with the solution for the horizontal force between misaligned plates.

5.5 Electrostatic Actuators

One of the few application areas for large electrostatic devices is in acoustics, where electrostatic speakers, headphones, and microphones provide what are generally regarded as the highest quality sound recording and reproduction components available.

To produce a loud low frequency sound requires the movement of a large volume of air, and moving it efficiently requires a structure with a large area. The force produced in a parallel plate capacitor is idealy suited to this, since it provides a uniform force across the entire surface of the plate so that the driving element can be very light and does not have to be very stiff.

Connecting a structure like Figure 5.4 directly to an acoustic signal will not be very satisfactory. Since the force $F = \frac{\epsilon_0 A}{2}\frac{v^2}{d^2}$ is always positive (tending to draw the plates together), the first time a signal was applied the plates would become stuck together.



This is easily corrected by providing an elastic restoring force $F = -kx$ to force them back apart. This could be done either by placing an elastic dielectric between the plates, by tensioning the movable plate (like a drum head) or by sealing the space between the plates. This still leaves us with the problem that since the force is proportional to the *square* of the applied voltage, severe distortion will result.

This distortion can be substantially reduced by *biasing* or *polarizing* the plates by adding a large DC voltage to the AC signal voltage. In this case we have

$$F = \frac{\epsilon_0 A}{2(d-\varepsilon)^2}(V_0 + v)^2$$

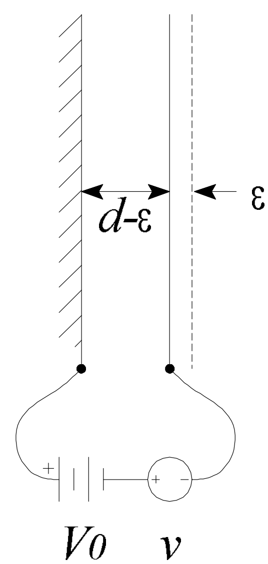
which for $d \gg \varepsilon$ gives

$$F \approx \frac{\epsilon_0 A}{2d^2}({V_0}^2 + 2V_0 v + v^2)$$

and if $v \ll V_0$

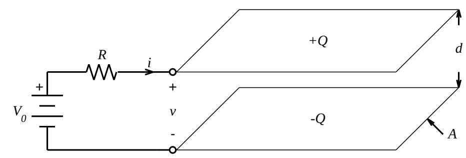
$$F \approx \frac{\epsilon_0 A}{2d^2}({V_0}^2 + 2V_0 v)$$

As the bias voltage is increased, both the linearity and the sensitivity will improve.



5.6 Electrostatic Sensors

A device like that in Figure 5.3 could be used as a microphone to sense acoustic pressure waves. However, rather than explicitly sensing changes in the capacitance, it is made self generating.



The plates are connected to a large polarizing voltage V_0 through a large resistor R. If the at rest spacing of the plates is d_0, then the at rest capacitance will be $C_0 = \frac{\epsilon_0 A}{d_0}$. In combination with R this forms an RC circuit with time constant $\tau = RC_0$. At rest, the charge on the plates will be $Q_0 = C_0 V_0$. If $\tau \gg 1/f_{min}$ where f_{min} is the lowest frequency present in the acoustic signal, then the charge on the plates will not change significantly during the course of a cycle, i.e. $Q \approx Q_0$.

But since the spacing is changing in response to changes in acoustic pressure, we have $d = d_0 + \varepsilon(t)$. The voltage at the plates is then

$$v = \frac{Q}{C} \approx \frac{Q_0}{\epsilon_0 A/(d_0 + \varepsilon(t))} = \frac{Q_0}{\epsilon_0 A}(d_0 + \varepsilon(t)) = V_0 + \frac{Q_0}{\epsilon_0 A}\varepsilon(t)$$