Chapter 6

Magnetics

6.1 The B Field and the Magnetic Force

If the electric field is zero, and the charge density is unchanging, the Lorentz force law becomes:

$$\mathbf{F} = q\mathbf{u} \times \mathbf{B} \tag{6.1}$$

i.e. the magnetic field describes the force exerted on moving charge. Note that we can have *motion* of charge in a closed circuit without the *charge density* changing. This is the case when current is flowing in a conductor, in which case 6.1 becomes

$$d\mathbf{F} = i d\mathbf{l} \times \mathbf{B} \tag{6.2}$$

where $d\mathbf{F}$ is the force acting on the portion of the conductor of length $d\mathbf{l}$ through which a current i is flowing.

6.1.1 Finding the Magnetic Field

We can parallel our investigation of the **E** field by finding the force on very simple currents and generalizing the results.

Ampère's Force Law Two parallel current carrying conductors are drawn together if the currents are in the same direction, or spread apart if they are in opposite directions. The magnitude of the force on a length L of one of the conductors is

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}L \tag{6.3}$$

Following our development of the electric field, we get the force per unit length on conductor

1 due to the *magnetic field* of current 2:

$$\frac{F}{L} = I_1 \frac{\mu_0}{2\pi} \frac{I_2}{r} = I_1 \mathbf{B_2} \tag{6.4}$$

Equation 6.4 is for the special case of parallel conductors. Because current is a vector quantity, it is necessary to take into account its direction as well as its magnitude. For the general case of arbitrary currents, the force on an element of conductor 1 is

$$d\mathbf{F} = \frac{\mu_0}{4\pi r^2} I_1 I_2 [d\mathbf{l_1} \times (d\mathbf{l_2} \times \mathbf{a_r})]$$

or

$$d\mathbf{F} = I_1 d\mathbf{l_1} \times \mathbf{B}_2 \tag{6.5}$$

where

$$\mathbf{B_2} = \frac{\mu_0}{4\pi} \oint_{C_2} \frac{I_2 d\mathbf{l_2} \times \mathbf{a}}{r^2} \tag{6.6}$$

and C_2 is the path of current I_2. 6.5 is identical to 6.2, i.e. a form of the Lorentz force law, and 6.6 is known as the *Biot-Savart law.* The units of B are the tesla (T) which from Equation 6.5 has the dimensions N/Am.

Again, as in the case of the Electric field, we may draw field lines whose direction is that of **B** and whose closeness represents its magnitude. In this graphical representation, the field lines are called *lines of flux*, hence **B** is usually called the *magnetic flux density.*

The magnetic field lines due to a straight conductor are circular, as shown in Figure 6.1. The direction of the lines is given by the *right hand rule* (Figure 6.2): "If the right hand

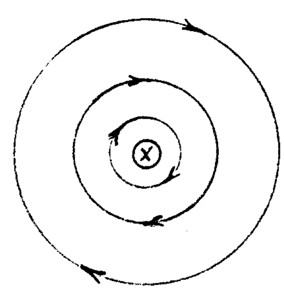


Figure 6.1: Magnetic field of a straight wire.

is wrapped around the conductor with the thumb pointing in the direction of current flow, the fingers will point in the direction of the magnetic field".

If we form the wire into a loop, we get the field shown in Figure 6.3a. Figure 6.3b shows the field of a series of loops or *solenoid.*

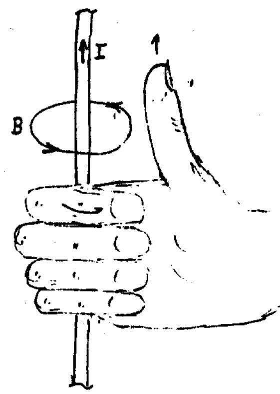


Figure 6.2: The right hand rule.

6.1.2 Magnetic Materials

The relationships developed so far describe the behavior of the magnetic field in a vacuum. In the presence of matter, the relationship between applied currents and the resulting **B** field given by (6.6) no longer holds. For most materials we find a slight decrease or increase in the observed field compared to the expected value, but the relation is still a linear function of current. Such materials are called *diamagnetic* and *paramagnetic* respectively.

A third class of magnetic materials, called *ferromagnetic*, shows drastically different behavior:

1. There is a very large increase in **B** for the same current.
2. The relation between **B** and I is non linear.
3. A **B** field can exist in the absence of current.

We can accommodate this behavior by defining a magnetizing field **H** (called the *magnetic field intensity*) which depends only on the observable currents in the system (*Ampère's Law*):

$$\oint \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{ds} \tag{6.7}$$

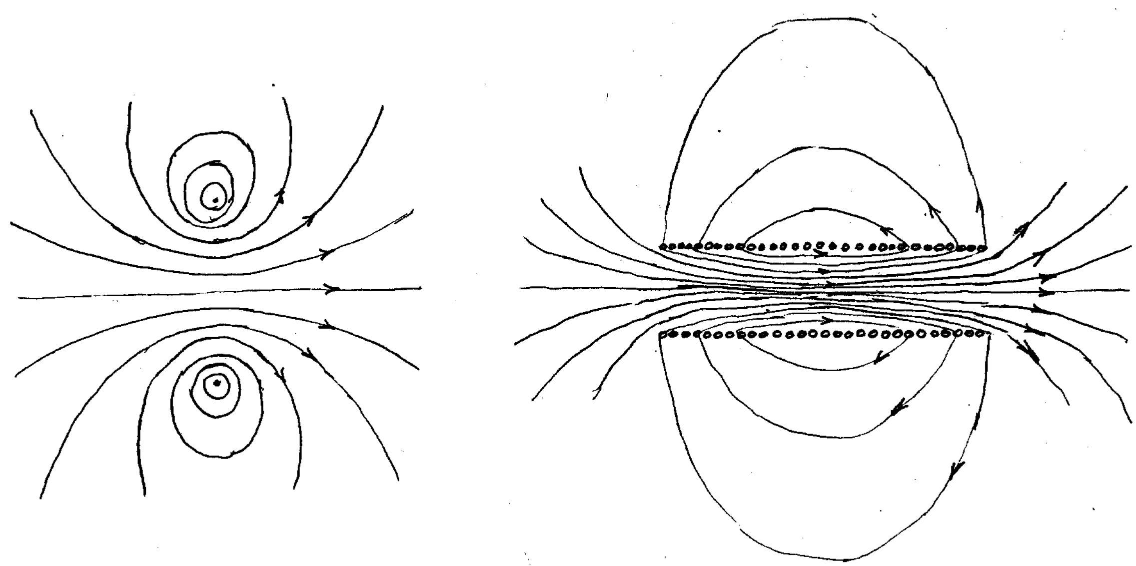
i.e. the line integral of **H** around any closed path is equal to the current flowing through the surface bounded by that path. We may also rewrite the Biot-Savart law in terms of **H**:

$$\mathbf{H} = \frac{1}{4\pi} \oint \frac{I\mathbf{dl} \times \mathbf{a_r}}{r^2} \tag{6.8}$$

or

$$d\mathbf{H} = \frac{I\mathbf{dl} \times \mathbf{a_r}}{4\pi r^2} \tag{6.9}$$

Unlike (6.6), (6.8) is valid both in free space and inside matter.



(a)

(b)

Figure 6.3: (a) Magnetic field of a single loop. (b) Magnetic field of a solenoid.

For materials whose behavior is linear, we may write

$$B = \mu H$$

where μ is the *permeability* of the material. For convenience, we usually refer to the *relative permeability* μ_r where

$$\mu = \mu_r \mu_0$$

Section A.4.3 gives the value of μ_r for some typical materials.

The source of the additional magnetism inside matter is the *bound currents* due to electron spin and electronic orbital motion. We will not consider the details of these phenomena, but will occasionally utilize bound currents in later analysis.

6.1.3 Flux and Flux Linkage

We may define the *magnetic flux* through a surface as

$$\mathbf{\Phi} = \int_S \mathbf{B} \cdot d\mathbf{S} \tag{6.10}$$

The unit of magnetic flux is the *weber* (Wb).

Magnetic field lines are often referred to as *lines of flux*. The association of the "number of lines of flux" in an area with the magnitude of the **B** field (magnetic flux density) provides a mnemonic for the interpretation of magnetic field diagrams. Although flux is in fact a scalar quantity, it is taken to have the direction of **B**. Unlike the electric field lines, which initiate and terminate on charge, magnetic flux lines are closed curves.

Flux Linkage In magnetic systems involving conducting loops (coils), the number of turns in the coil is an important factor. We define the *flux linkage*, λ, of a coil as the product of the number of turns in the coil and flux passing through, or *linking*, the coil:

$$\lambda = N\Phi \tag{6.11}$$

6.1.4 Magnetic Circuits

Although the currents that give rise to (non-permanent) magnetic fields are confined in one dimensional conductors, the fields themselves are distributed throughout a region of space. However, the fact that magnetic field lines are closed suggests the possibility of trying to marshal as much of the flux generated by a current as possible into a compact *magnetic circuit*. The means for achieving this spatial concentration of flux is to build structures of ferromagnetic material. This will allow us to approximate the distributed parameter, vector magnetic field as a lumped parameter, scalar magnetic circuit.

Consider the magnetic circuit in figure 6.4. The coil has N turns and is wound on a *core* of

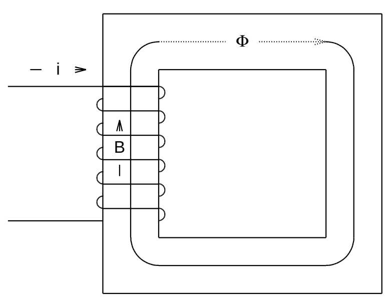


Figure 6.4: A magnetic circuit.

permeability μ, cross sectional area A, and mean length l_c.

If $\mu \gg \mu_0$, then we may assume that most of the flux produced by i is confined to the core, i.e. that B outside the core is negligible compared to B inside.

We can define the following terms:

Magnetomotive Force If we apply Ampère's law to the path defined by the center of the core,

$$H_c l_c = \oint \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = Ni \tag{6.12}$$

We define

$$\mathcal{F} = Ni \tag{6.13}$$

to be the *magnetomotive force* (mmf) produced by current i. This mmf gives rise to a magnetic field intensity H_c, which in turn produces a magnetic flux density B_c in the core.

Flux The flux density, B_c in the core is

$$B_c = \mu H_c$$

If we assume B is uniform across the core, then

$$\Phi_c = B_c A \tag{6.14}$$

is the total *flux* in the core.

Reluctance From (6.12)

$$H_c = \frac{Ni}{l_c} = \frac{\mathcal{F}}{l_c}$$

Combining this with (6.14)

$$\Phi = BA = \mu H_c A = \frac{\mu A}{l_c}\mathcal{F}$$

Defining the *reluctance*

$$\mathcal{R} = \frac{l_c}{\mu A}$$

we have

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \tag{6.15}$$

which is the equivalent of Ohm's law for magnetic circuits. We may also define *permeance* ($\mathcal{P}$) as the reciprocal of reluctance.

6.1.5 Faraday's Law and Induced Voltage

Magnetic force acts on moving charge. In the case of a current flowing in a magnetic field, this gives rise to a force on the conductor as given in (6.5). In the case of a conductor moving with respect to a magnetic field, the force acts on the charge carriers in the conductor to produce an *induced emf.*

Faraday's Law For a closed path S, *Faraday's law* states:

$$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \tag{6.16}$$

If S is a conductive loop the left hand side of (6.16) is the induced emf. For a closed circuit, the enduced emf will produce a current. If the circuit is opened, a voltage e will appear across its ends. The right hand side is the time derivative of the magnetic flux as defined in (6.10). So for a single turn conductive loop in a magnetic field:

$$e = -\frac{d\phi}{dt}$$

For a coil of N turns,

$$e = -N\frac{d\phi}{dt} = -\frac{d\lambda}{dt} \tag{6.17}$$

Note that Faraday's law applies both to time varying flux in a stationary coil ("transformer emf") and to flux change due to a conductor moving through a static flux ("motional emf").

The choice of sign for e is the source of some confusion. We will usually treat (6.17) as if it had no sign and use Lenz's law to provide the correct sign.

Lenz's Law The minus sign in the above equations is a manifestation of *Lenz's law* which states that the polarity of an induced emf e, caused by a change in the flux ϕ, will be such that it would produce (in a closed circuit) a current i, in a direction that would produce a field B, which would oppose the change in flux that produced the emf in the first place.

6.1.6 Inductance and Energy Storage

Consider the magnetic circuit of Figure 6.4 as a component of an electrical circuit. A current i, is flowing in the coil, producing the field B. If i varies with time, then so will the flux.

From (6.15) and (6.13):

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}}$$

so the flux linkage, λ is

$$\lambda = N\Phi = \frac{N^2 i}{\mathcal{R}} = \frac{N^2 i \mu A}{l}$$

We define the *inductance*

$$L = \frac{\lambda}{i} \tag{6.18}$$

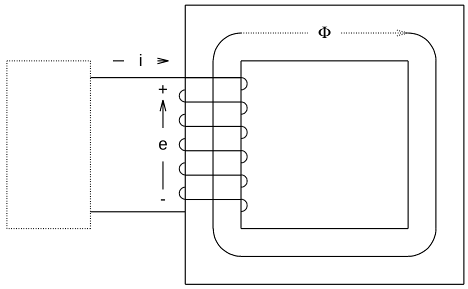


Figure 6.5: A simple inductor.

as the ratio of the flux linkage to the current which produces it. If the current, and hence the flux linkage, is changing with time, a voltage, e is induced in the coil:

$$e = \frac{d\lambda}{dt}$$

Because the sign convention for loads is different from that for sources (current flows *out* of the positive terminal of a source, but *into* the positive terminal of a load), the sign for e is positive.

Combining this with (6.18) we get

$$e = \frac{dLi}{dt} = L\frac{di}{dt} \tag{6.19}$$

Energy Storage If we assume a lossless system, then the rate at which energy is added to an inductor is

$$p = ei$$

so the increment in stored energy is

$$dW = p\,dt = ei\,dt = \frac{d\lambda}{dt}i\,dt = i\,d\lambda$$

and the total stored energy is

$$W = \int_0^{\lambda_f} i\,d\lambda = \int_0^{\lambda_f} \frac{\lambda}{L}\,d\lambda = \frac{{\lambda_f}^2}{2L} = \frac{1}{2}L{i_f}^2 \tag{6.20}$$

where λ_f is the final flux linkage, and i_f is the final current.

6.2 Ferromagnetic Materials

As mentioned in Section 6.1.2 most materials have relative permeabilities very close to one. The materials of greatest interest for electromagnetic devices are the ferromagnetic materials having permeabilities much greater than one. These materials have two other significant properties in addition to their high permeability: the relation between the magnetic intensity H and the resulting magnetic flux density B is nonlinear, and a non-zero B can exist in the absence of external H.

6.2.1 Magnetic Moment

The origin of the magnetic properties of matter is in the orbital and spin motion of the electrons of its constituent atoms. We may regard the electron spin and electron orbital motion as small loops of circulating current. Such a loop will produce a magnetic field proportional to the current and to the area of the loop. We define the *magnetic moment* $\mathbf{p_m}$ of a current loop as

$$\mathbf{p_m} = \mathrm{iA}\mathbf{a_n} \tag{6.21}$$

where A is the area of the loop and $\mathbf{a_n}$ is normal to the plane of the loop with direction given by the right hand rule (Figure 6.6). Both the spin and orbital magnetic moments of

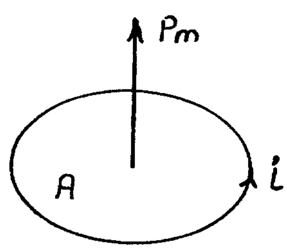


Figure 6.6: Magnetic moment.

electrons occur in multiples of the *Bohr magneton*, μ_B, 9.27×10^{-24} Am^2.

If a magnetic moment p_m is placed in a magnetic field of flux density B, it will experience a torque

$$\mathbf{T} = \mathbf{p_m} \times \mathbf{B} \tag{6.22}$$

which tends to rotate the moment toward alignment with the field (Figure 6.7).

6.2.2 Magnetization

In many materials symmetry or randomness of orbit and spin orientation result in a net cancellation of magnetic moment. In ferromagnetic materials, unbalanced electron spin results in a large magnetic moment per atom (2.21 μ_B for iron, 1.72 μ_B for cobalt, and 0.6

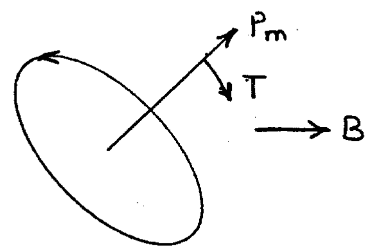


Figure 6.7: Torque on a magnetic moment in a magnetic field.

μ_B for nickel). In addition, the crystalline structure favors alignment of the moments of adjacent atoms along crystal axes.

The individual magnetic moments are ordinarily random but will tend to align with an external field as indicated by (6.22). The degree of difficulty required to achieve alignment varies depending on the orientation of the region and the history (previous alignment) of the region. Once all the moment in the sample is aligned, it can provide no further increase in the total magnetization, and the sample is said to be *saturated*. These effects result in the non-linear relationship between applied H and resultant B described in Section 6.2.3.

To determine the net magnetic effect of the aligned moment, consider the moment of each atom to be equivalent to a current i_M circulating around the volume d^3 (Figure 6.8(a)). For each atom

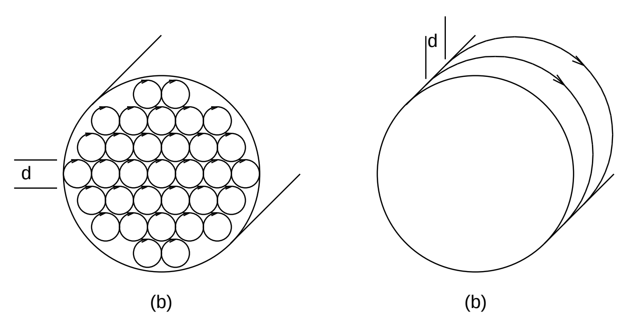


Figure 6.8: Magnetization of ferromagnetic rod. (a) Individual moments. (b) Equivalent Amperian current.

$$p_M = i_M d^2$$

Except at the surface, each edge of a loop is adjacent to an edge of opposite direction, so that the internal currents cancel, and the net effect is that of a current of i_M on the surface.

Since there is one of these loops for each length d of the material, the equivalent magnetic field intensity in the material is

$$H_{equiv} = \frac{i_M}{d} = \frac{i_M d^2}{d^3} = \frac{p_M}{d^3} = M$$

i.e. the magnetic moment per unit volume. We define the net aligned magnetic moment per unit volume as the *magnetization* M.

The total magnetic flux density in the material will be that due to the exciting coil plus the effect of the aligned magnetic moment:

$$B = \mu_0(H + M) = \mu_0(H + \chi H) = \mu_0(1 + \chi)H = \mu_0\mu_r H$$

where χ is the *magnetic susceptibility* of the material, which represents the magnetic field enhancement in response to the applied field H.

6.2.3 Magnetization Curves

Because of the complex relation between B and H for a ferromagnetic material, it is not possible to give a single value for the permeability. This relationship is best represented in a *magnetization curve* or *B-H curve*. If we take a piece of completely demagnetized material and subject it to a slowly increasing magnetic intensity H, the flux density B increases in a manner similar to that shown in Figure 6.9.

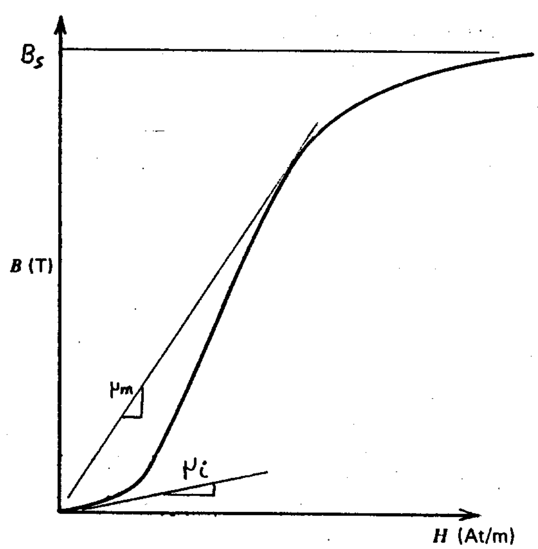


Figure 6.9: Initial magnetization curve.

The most commonly used value for permeability is the *amplitude permeability* which is simply the ratio of B to H at any point on the curve. The *maximum permeability* is the maximum value of the amplitude permeability, and is equal to the slope of the line from the origin tangent to the upper knee of the B-H curve.

If we are interested in changes in B as a function of changes in H, it is necessary to consider the *differential permeability* μ_d defined as the slope of the magnetization curve at a given point:

$$\mu_d = \frac{dB}{dH}$$

The slope of the magnetization curve at the origin is called the *initial permeability* :

$$\mu_i = \lim_{H \to 0} \frac{B}{H}$$

Although we have given the definitions of the various permeabilities as absolute permeabilities, data are often given as relative permeabilities, e.g.

$$\mu_r = \frac{1}{\mu_0} \frac{B}{H}$$

When in doubt, just look at the size of the number: for a ferromagnetic substance, if it is on the order of one, it is probably an absolute permeability; if on the order of 1000, a relative permeability.

Saturation As the magnetic intensity H is increased, the flux density reaches a level B_s called the *saturation flux density* above which any additional increase in B is essentially due to free space permeability, i.e. the material itself is completely magnetized or *saturated*.

Residual Magnetism If after reaching saturation, we reduce the magnetizing force H, B does not follow the same path it did as H was increased. As shown in Figure 6.10 the flux does not return to zero as H is decreased to zero. Instead a *residual flux density* or *remanent magnetization* B_r remains in the material.

If we apply magnetic intensity in the opposite direction to that which produced the magnetization, B decreases until a value of H_c is applied which reduces the flux to zero. H_c is called the *coercive force* or *coercivity*. Materials with a low value of H_c are termed magnetically *soft* in contrast to *hard* materials having a high H_c suitable for permanent magnets.

Hysteresis If we continue reversing the magnetic intensity between plus and minus H_s, we reach a steady state *symmetrical hysteresis loop*, as shown in Figure 6.11.

The area inside the loop is related to the energy required to reverse the magnetization as the magnetizing force is reversed. This energy is nonrecoverable.

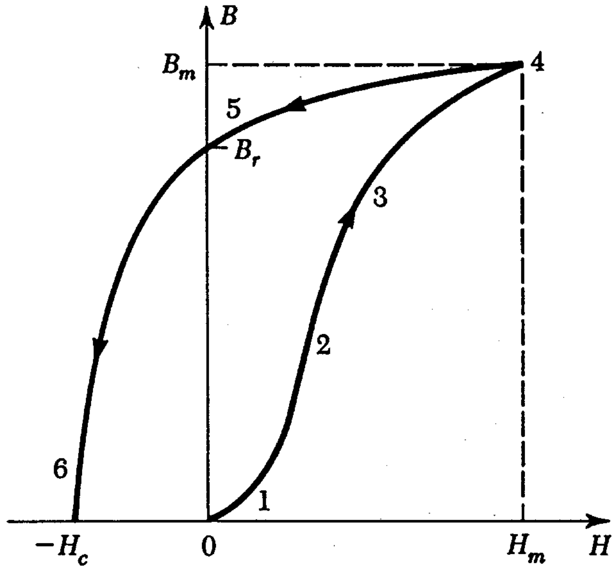


Figure 6.10: Residual magnetic flux and coercive intensity.

6.2.4 Energy in a Magnetic Field

The process of establishing a magnetic field in a piece of iron (or other magnetic material) requires energy. The circuit in Figure 6.12 consists of N turns of wire on a toroidal core of mean circumference l and cross sectional area A. At any time, the electrical power entering the coil is:

$$p = vi = Ri^2 + i\frac{d\lambda}{dt}$$

The first term is the power dissipated as heat in the resistance of the coil. The second term is the power flowing into the energy of the magnetic field. Representing this by p_B, and using (6.10), (6.11), and (6.12) we have:

$$p_B = i\frac{d\lambda}{dt} = Ni\frac{d\Phi}{dt} = Ni\,A\frac{dB}{dt} = HlA\frac{dB}{dt}$$

The total stored energy for a flux density B is

$$W_m = \int p_B dt = \int_0^B lAH\,dB = V\int_0^B H\,dB$$

where V is the volume of the core. This gives an *energy density* (magnetic energy per unit volume) of

$$w_m = \frac{W_m}{V} = \int_0^B H\,dB \tag{6.23}$$

If we assume the material has constant permeability μ,

$$w_m = \int_0^B \frac{B}{\mu}\,dB = \frac{B^2}{2\mu} = \frac{1}{2}BH = \frac{1}{2}\mu H^2$$

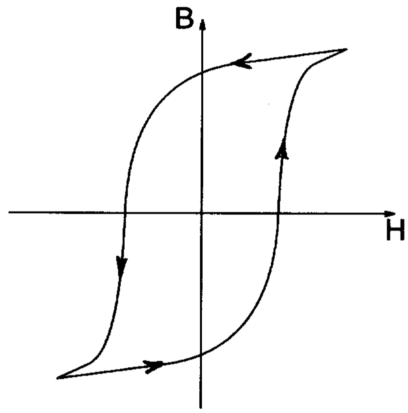


Figure 6.11: A typical hysteresis loop.

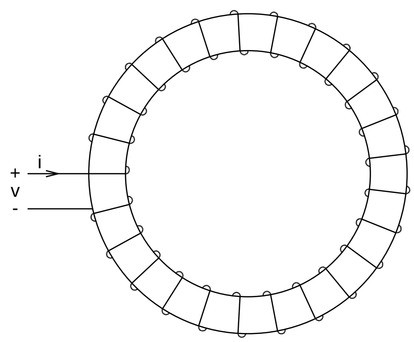


Figure 6.12: Toroidal magnetic circuit.

6.2.5 Magnetic Losses

Magnetic circuits with time varying flux are subject to two loss mechanisms: *eddy currents* and *hysteresis loss*.

Eddy currents A time varying magnetic field in the core gives rise to induced electric fields according to Faraday's law (Figure 6.13). In a magnetic material that is electrically conductive, these fields result in *eddy currents* which circulate in the core material. These currents combined with the resistivity of the core material produce I^2R heating losses.

Eddy current losses are proportional to the square of the frequency and the square of the peak flux density. They may be reduced by increasing the resistivity of the material by

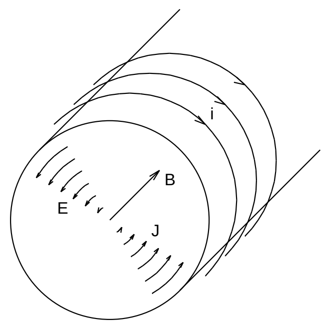


Figure 6.13: Eddy currents.

alloying or using non-conductive materials (e.g. ferrites), or by reducing the area of the current loops by using thin insulated laminations or powder in a non-conductive binder.

Hysteresis Loss Assume the circuit of Figure 6.12 has the hysteresis loop shown in Figure 6.14, and that we are cycling the magnetic field intensity between H_1 and $-H_1$.

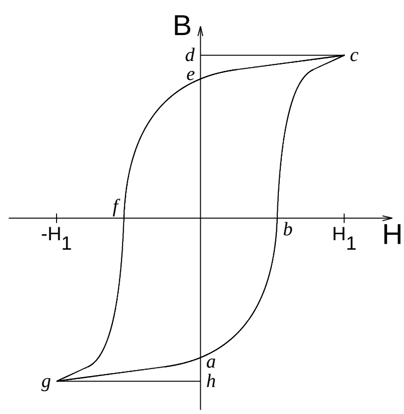


Figure 6.14: Hysteresis loss.

From (6.23) the energy per unit volume put into the magnetic field in changing the flux density from point a to point c is:

$$\Delta w = \int_{B_a}^{B_c} H \, dB$$

Graphically this is represented by the area between the B-H curve and the B axis, i.e. the area *abcda*.

When H is reduced from H_1 to zero, the flux density decreases to B_e and the energy per unit volume released by the field and returned to the coil and the source is the area *cdec*. The negative excursion produces a similar result.

In a complete cycle, an amount of energy per unit volume equal to the area of the loop is put into the circuit, but not recovered. This "missing energy" is the *hysteresis loss* and is dissipated as heat.

6.3 Magnetic Circuits

In Figure 6.4, the magnetic circuit was contained entirely within the magnetic material. To produce useful devices we must produce strong magnetic fields in air gaps, either to allow the introduction of a current carrying conductor, or to allow relative motion between parts of the device.

Figure 6.15 shows a simple circuit having an air gap.

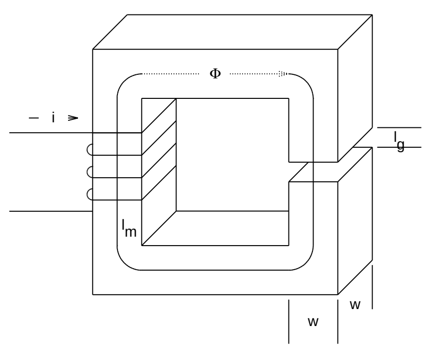


Figure 6.15: Magnetic circuit with air gap.

If $g \ll w$ then we can ignore fringing and assume the effective area of the gap is equal to the cross sectional area of the core. As the size of the gap becomes significant, a standard approximation for the effect of fringing is to add the gap length to each of the dimensions making up its cross sectional area.

If we neglect fringing, average flux density in the core and the air gap is

$$B = \frac{\phi}{A}$$

To support the flux density B in the air gap, the required magnetic field intensity H_g is

$$H_g = \frac{B}{\mu_0}$$

A much smaller field intensity H_m is required in the core to produce the same flux density. From Ampère's law

$$H_m l_m + H_g l_g = Ni$$

We may state this in terms of the magnetic circuit parameters, $\mathcal{R}$, Φ, and $\mathcal{F}$:

$$\Phi(\mathcal{R}_m + \mathcal{R}_g) = \mathcal{F}$$

i.e. the total reluctance of a series circuit is the sum of the reluctances of its parts.

Idealized Circuits For most circuits of interest, μ_m will be several orders of magnitude more than μ_g. In this case we may neglect the term $\mathcal{R}_m$. This is similar to an electric circuit where the conductors have such low resistance compared to other circuit elements that it is ignored, i.e. conductors are assumed to have zero resistance. We will often model magnetic circuits in a similar fashion, by assuming that the core has zero reluctance (i.e. the core material has infinite permeability).

Other effects which are ignored in this idealized model are

- nonlinearity of B-H curve
- saturation
- leakage and fringing flux

Boundary Relations It can be shown that at the boundary between two regions of different permeability (having no current density on the boundary surface) the normal component of B and the tangential component of H are continuous, i.e.

$$\begin{aligned} B_{1n} &= B_{2n} \\ H_{1t} &= H_{2t} \end{aligned} \tag{6.24}$$

If we take region 1 to be air ($\mu \approx \mu_0$) and region 2 an ideal magnetic material ($\mu \to \infty$), then within region 2

$$H_{2t} = \frac{B_{2t}}{\mu_2} \to 0 \quad (\mu_2 \to \infty)$$

i.e. the tangential component of the field intensity is zero in an ideal material. From (6.24) the tangential component in air is also zero. This implies *lines of flux are perpendicular to the surface of a perfect magnetic conductor.*

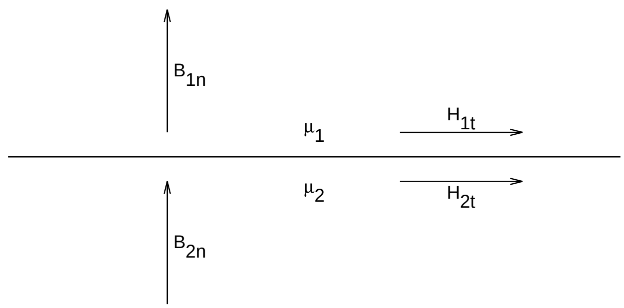


Figure 6.16: Boundary between two magnetic materials.

6.4 Permanent Magnets

In the circuit of Figure 6.15, the magnetic flux was generated by the current flowing in a coil. We may also create flux by placing a piece of permanently magnetized material in the circuit. Figure 6.17 shows such a circuit, consisting of a permanent magnet, soft iron core, and an air gap. We could magnetize the material by filling the gap with a soft iron

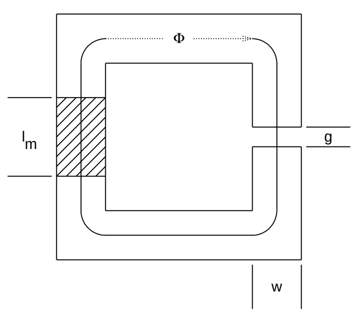


Figure 6.17: Magnetic circuit with permanent magnet.

"keeper", winding a temporary coil around the core, and applying a sufficiently large pulse of current to drive the material into saturation.

What happens when we remove the keeper and restore the gap? Applying the circuital law around the circuit gives

$$H_m l_m + H_c l_c + H_g l_g = 0$$

where l_c is the effective length of the core material. Assuming μ_c to be infinite,

$$H_m = -H_g \frac{l_g}{l_m} \tag{6.25}$$

Since the flux around the path is continuous, we must have $B_m A_m = B_g A_g$ or

$$B_m = B_g \frac{A_g}{A_m} \tag{6.26}$$

Combining (6.25) with $B_g = \mu_0 H_g$ and substituting into (6.26) we have the following relation between the flux density and field intensity in the material:

$$B_m = -\mu_0 \frac{A_g}{A_m} \frac{l_m}{l_g} H_m \tag{6.27}$$

Since B_m and H_m must also satisfy the relation given by the B-H curve, we may find the operating point by finding the intersection of the load line (6.27) with the B-H curve as shown in Figure 6.18. When the keeper is removed after magnetizing the material, the

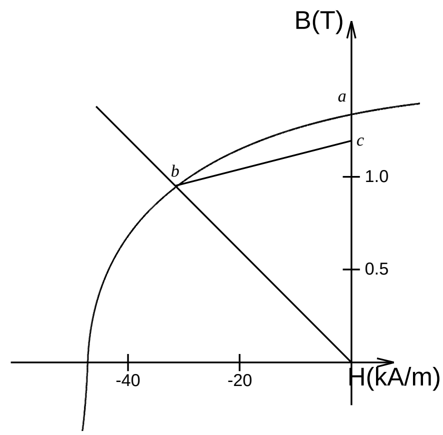


Figure 6.18: Second quadrant B-H curve (Alnico V).

operating point moves along the path *ab*. If the keeper is reinserted, the operating point moves along the *recoil line bc*.

In designing a magnetic circuit, the air gap flux density is usually specified, and it is necessary to determine the required size for the magnetic material. Using (6.25) and (6.26)

$$\begin{aligned} V_m &= A_m l_m \\ &= \left(\frac{B_g A_g}{B_m}\right)\left(\frac{-H_g l_g}{H_m}\right) \\ &= \frac{{B_g}^2 V_g}{\mu_0 \mid B_m H_m \mid} \end{aligned} \tag{6.28}$$

where V_m is the volume of magnetic material, and V_g is the volume of the air gap. The product $B_m H_m$ is the *energy product* of the material at the operating point. The volume of magnetic material required may be minimized by choosing the operating point to be at the maximum energy product of the material.