Chapter 7

Magnetic Devices I

7.1 Moving Copper: BIL and BLU

BIL If a conductor of length L carrying a current I is placed in a uniform magnetic field B which is perpendicular to the length of the conductor, then by 6.2 each length $d\mathbf{L}$ experiences a force $d\mathbf{F} = Id\mathbf{L} \times \mathbf{B}$. The total force on the conductor is $F = BI\int_0^L dL = BIL$ and is mutually perpendicular to the conductor and the field.

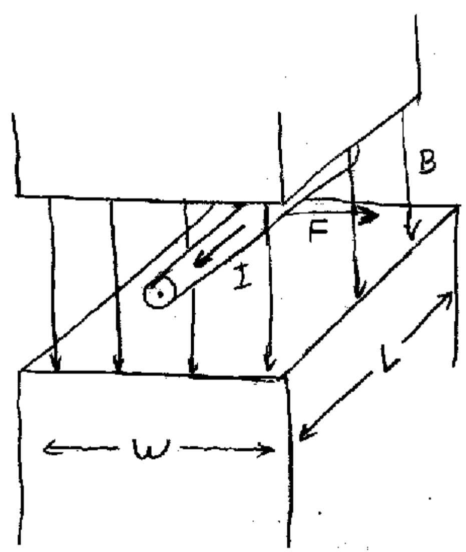
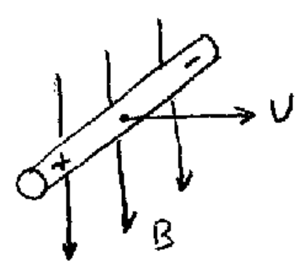
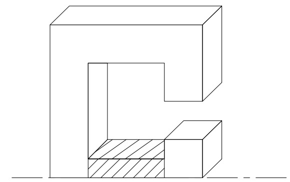


Figure 7.1: $F = BIL$

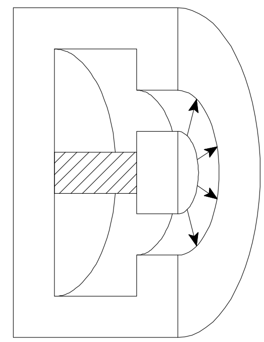
BLU If a conductor of length L is moving at a velocity U in a uniform magnetic field B in a direction perpendicular to both the field and the conductor, then by 6.1 each charge carrier experiences a force $f = q_e\mathbf{U} \times \mathbf{B} = BUq_e$ along the conductor. This is the same force it would experience due to an electric field $E_{equiv} = BU$ parallel to the conductor, i.e. a *motional emf* of $e = E_{equiv}L = BLU$ is produced in the conductor.



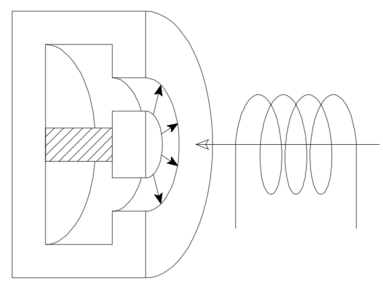
The uniform field is usually produced in a magnetic circuit like that in Figure 6.15. For simplicity, we will use a permanent magnet and will place it in the lower limb of the structure.



Now rotate the core about the indicated axis. This produces a structure having a cylindrical air gap with the field directed radially across the gap.

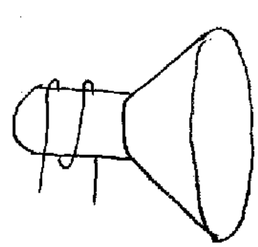


Instead of the length over which the force can act being restricted to the width of the gap, we can place a coil with a number of turns, N, each having a length equal to the circumference of the gap. The total force produced is $F = 2\pi rNBI$. If the coil moves with a velocity u, then an emf of $e = Blu = 2\pi rNBu$ will appear across the ends of the coil.



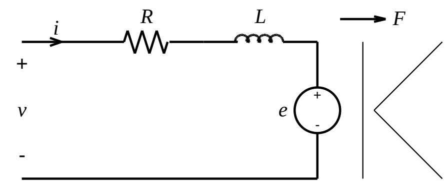
7.2 The Voice Coil

This device is called a *voice coil* because its initial application was as the driver for a loudspeaker. For this application, a *cone* is attached to the coil to couple its motion to the air.



7.2.1 Loudspeaker Dynamics

Suppose that instead of driving the coil with a current, we drive it from a voltage source, as shown in the figure. Here R is the resistance of the coil and L is its inductance. Then using KVL we get



$$v = Ri + L\frac{di}{dt} + e$$

We can represent the acoustic load as an effective mass m_e of the air to which the cone is coupled and a *radiation resistance* r. Let m be the sum of the cone and the effective mass of the air. Then the mechanical force seen by the coil is

$$F = ru + m\frac{du}{dt}$$

We also have two *coupling equations*

$$\begin{aligned} F &= Bil \\ e &= Blu \end{aligned}$$

where l is the total length of the wire in the coil. Combining these with the electrical and mechanical differential equations, we get

$$\begin{aligned} v &= Ri + L\frac{di}{dt} + Blu \\ Bli &= ru + m\frac{du}{dt} \end{aligned}$$

Combining these we get

$$Lm\frac{d^2u}{dt^2} + (Rm + Lr)\frac{du}{dt} + (Rr + (Bl)^2)u = Blv$$

which we can solve for u, given v.

Alternately, we can take the Laplace transforms $I(s) = \mathcal{L}(i(t))$, $V(s) = \mathcal{L}(v(t))$, and $U(s) = \mathcal{L}(u(t))$, giving

$$\begin{aligned} V &= RI + LsI + BlU \\ BlI &= rU + msU \end{aligned}$$

Eliminating U

$$\begin{aligned} \frac{V}{I} &= R + Ls + Bl\frac{U}{I} \\ \frac{U}{I} &= \frac{Bl}{r + ms} \end{aligned}$$

gives us the electrical impedance seen by the source

$$Z(s) = \frac{V}{I} = R + Ls + \frac{(Bl)^2}{r + ms}$$

If we eliminate I

$$\begin{aligned} V &= RI + LsI + BlU \\ I &= \frac{rU + msU}{Bl} \\ V &= \frac{(R + Ls)(r + ms)U + (Bl)^2 U}{Bl} \end{aligned}$$

we get the transfer function

$$H(s) = \frac{U}{V} = \frac{Bl}{(R + Ls)(r + ms) + (Bl)^2}$$

relating the acoustic output to the electrical input.

7.2.2 Electrostatic vs. Magnetic Force

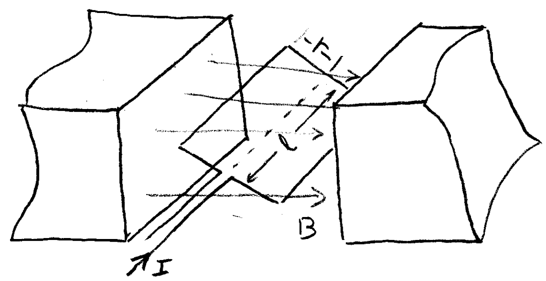
It is instructive to compare the magnitude of the forces produced by our electrostatic and magnetic loudspeakers. From Equation 5.15, the total force on a diaphragm 1 m square, separated from the fixed plate by 1 mm, and driven by 100 V is 0.044 N. For a magnetic speaker with a 1 cm coil having 10 turns working in a field of 1 T and driven by 1 A, the force produced is 0.31 N. Although the total force produced by each is comparable, the magnetic device concentrates its force in an area less than 1/10000 the size.

7.2.3 Dynamic (Magnetic) Microphone

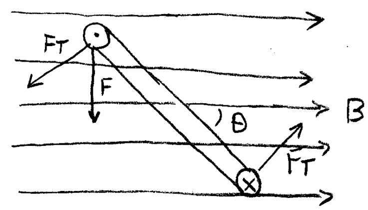
It is important to note that the motional emf (e above) is produced whether or not the loudspeaker is being driven electrically. If we place the speaker in an acoustic field, the motion of the air will be coupled via the cone into a motion of the coil. This motion will produce a voltage $v = Blu$ at the terminals. In other words, we can use a dynamic loudspeaker as a microphone. In fact, in most intercom systems the same device is used both as the speaker and the microphone depending on the position of the talk switch.

7.3 Rotary Moving Coil Actuators

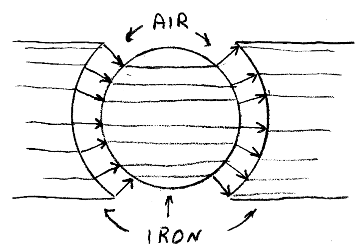
If we make the gap in Figure 7.1 wide enough, we can place a loop rather than a single length of wire into it. We will assume that the loop is constrained to rotate about an axis through the center of the gap, perpendicular to the field.



Because the current is flowing in opposite directions in the two legs of the loop, the force on the left side is downward while the force on the right side is upward. This results on a torque of $T = 2rBil\cos(\theta)$.



By contouring the faces of the pole pieces and placing a cylindrical core between them, a stronger, radially directed field is produced. In this case the torque will be $T = 2rBil$, independent of θ. If there are N turns in the coil, the torque will be $T = 2NrBil$.



The d'Arsonval meter movement If a torsion spring is connected to the coil, it will produce a torque proportional to θ, $T = -k\theta$. In the steady state, $2NrBil = -k\theta$ or $\theta = \frac{2NrBl}{k}i$, i.e. the angle of the coil will be proportional to the current. With a pointer and graduated arc to indicate the angle this formed the measuring element of nearly all DC current and voltage meters before the advent of the DMM.