Chapter 8

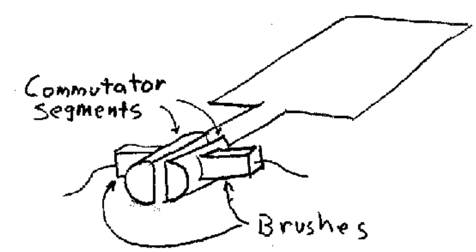
The DC Motor

8.1 Commutation

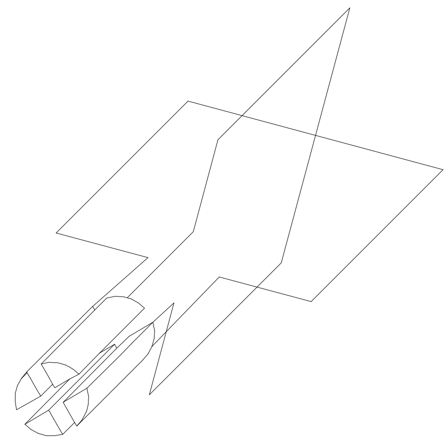
The rotary actuator in Section 7.3 has a useful range of something less than 180°: It produces a uniform torque while the coil is in the gap (in a "horizontal" plane), but the torque goes to zero when the coil reaches a vertical plane. If we rotate it past this torque null, the torque reverses direction, tending to restore the loop to the vertical plane. To make this device into a useful, continuous rotation motor, we need to (a) keep the torque pointing in the same direction, and (b) remove the region of zero torque.

Let's consider problem (a) first. If we could reverse the direction of the current when the coil crosses the vertical plane, then the reversal of the current would reverse the reversal of torque and the non-zero torque would always be in the same direction. If the load is sufficiently light, inertia will probably carry the coil through the torque null and we would have a motor with continuous rotation.

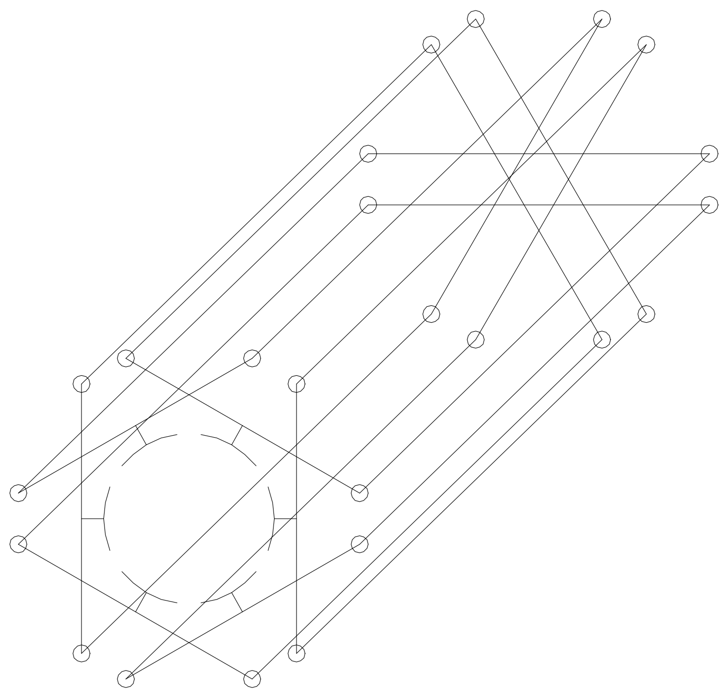
We can do this with a device called a *commutator* which is essentially a rotary switch connected to and hence synchronized with the coil. The stationary parts of the switch are called *brushes* and are usually made of carbon.



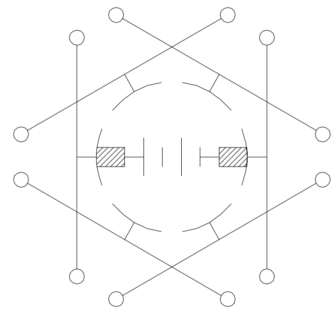
The problem of the torque null can be solved by adding a second coil so that when the first coil has left the gap, the second can continue to produce torque.



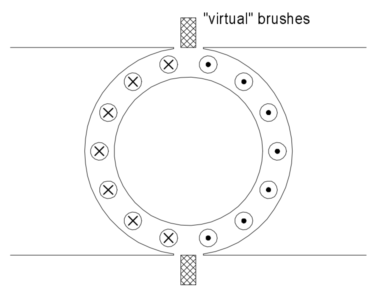
This will work, but is inefficient: only half of the wire is carrying current at any one time, and wire not carrying current is not producing torque. We can fix this by connecting the coils together in series, as shown here for a three coil winding with a six segment commutator.



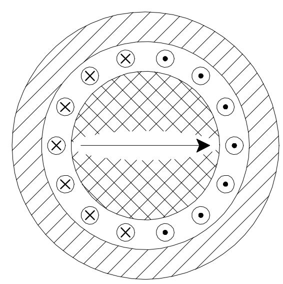
A cross sectional view makes it easier to see how the commutation works but makes it more difficult to see how the coils are connected on the far end.



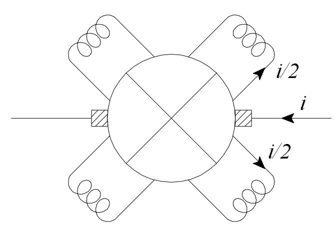
Note that although the brushes lie in a horizontal plane, the current in a coil changes direction when it crosses the vertical plane. For this reason it is often convenient to draw the brushes as though they contacted the coils directly at their periphery. In this case current switching occurs in the same plane as these “virtual” brushes.



We can also put the magnet on the inside of the coil, with a cylindrical iron shell on the outsite to complete the flux path.



To find the total torque produced by a motor, we can sum the contributions due to each turn. Because the total armature current I_a is divided between the two halves of the winding, each conductor is carrying a current of $I_a/2$.

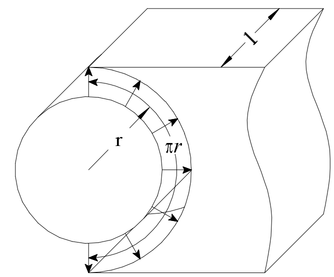


Each length l of this conductor in the gap experiences a force of $F = BlI_a/2$ and contributes a torque of $T = rF = rBlI_a/2$. Since each turn of the coil has two segments of length l in the gap, each turn contributes $2rBlI_a/2 = rBlI_a$. With N total turns we get

$$T = NrBlI_a = K_t I_a \tag{8.1}$$

where $K_t = NrlB$ is called the *torque constant* of the motor.

It is sometimes convenient to describe the torque in terms of the total gap flux $\Phi = \pi r l B$ Substituting $rlB = \frac{\Phi}{\pi}$ into 8.1 we get

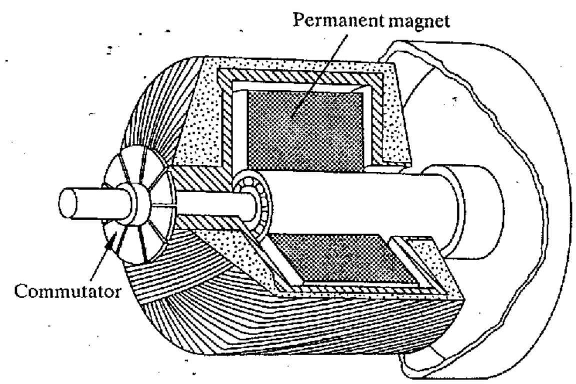


$$T = NRlBI_a = N\frac{\Phi}{\pi}I_a = \frac{N}{\pi}\Phi I_a = K_\Phi \Phi I_a = K_t I_a$$

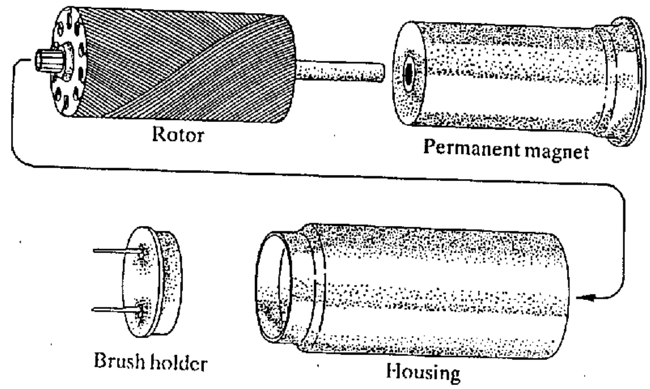
Where K_Φ is called the *motor constant* and is often written as simply K, i.e. $T = K\Phi I_a$.

8.2 Moving Coil Motors

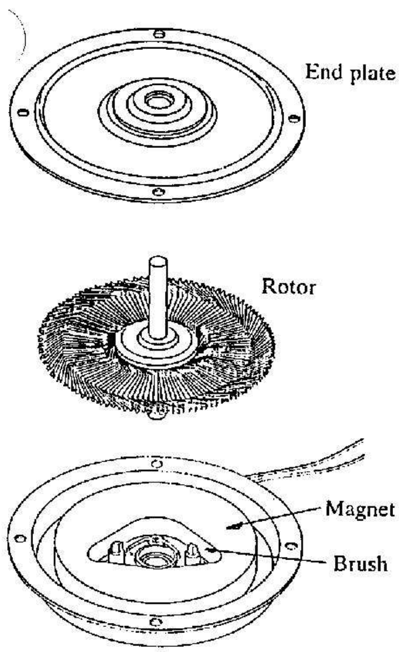
These cross sectional views hide the fact that the volume where the cylindrical pole piece or magnet must reside is completely surrounded by the coil winding. This makes construction difficult as the coil must be wound with this already inside. Nevertheless, motors are made using this structure and are called *ball wound motors* due to the resemblance of the coil to a ball of string.

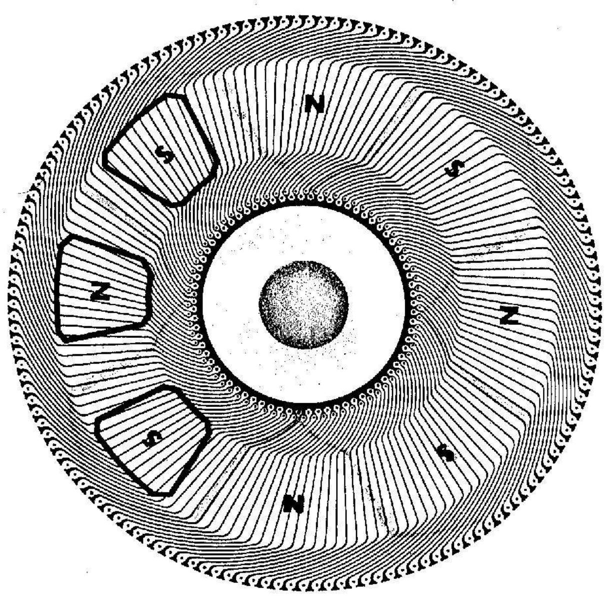


One way to make construction easier is to change the path of the rear portion of the coil, creating an open, cylindrical winding rather than a closed ball. This produces what is called a *cylindrical coreless motor*.



By going to the opposite extreme and compressing the cylindrical portion of the winding and leaving only the ends, we get what is called a *pancake motor*. Because the armature winding is often fabricated as a printed circuit rather than wound with wire, it is also often called a *printed circuit motor*.





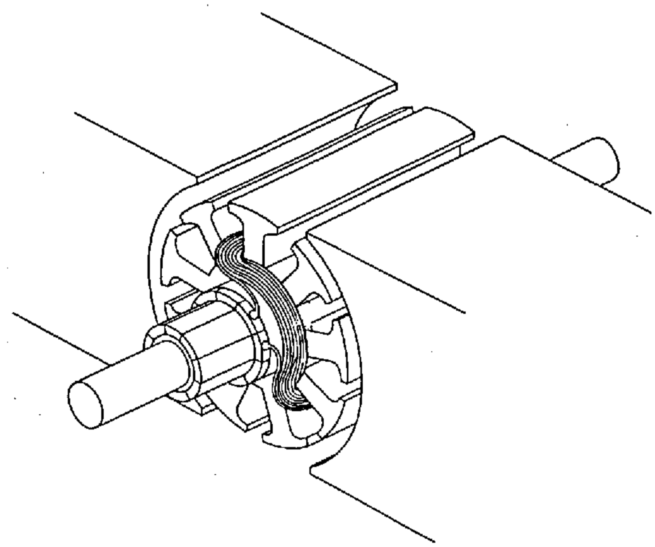
The class of motors where the moving member is made entierly of current carrying conductors (usually copper) and their supporting structure, but no magnetic material is called *moving coil motors*. Moving coil motors have a number of advantages: Their low moving mass gives them a low moment of inertia and hence good dynamic response. The fact that the coils do not enclose magnetic material gives them low inductance and low magnetic losses. The homogeneity of the winding reduces variations of torque with angular position (torque ripple).

However, they do have a few shortcomings: Since the torque producing windings must pass through the air gap, a large gap is required to accommodate a large current. However, a large gap requires a large, expensive magnet to produce a large **B** field. Because the torque is produced directly on the wire of the coils, the winding must be strengthened to withstand the forces and couple them to the shaft.

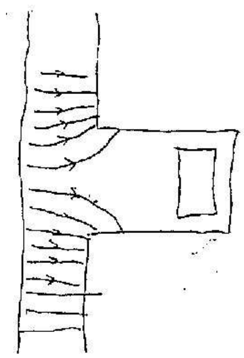
8.3 The Slotted Armature

The disadvantages noted for the moving coil motor were in fact insurmountable ones until the mid 20th century; the necessary high energy permanent magnet materials and high strength adhesives were simply not available. While waiting for these to be invented, makers of DC motors used a structure called the *slotted armature*.

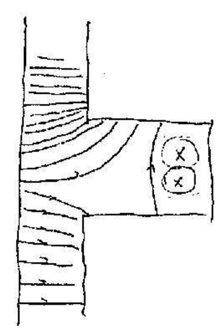
In this type of motor, the iron core rotates along with the windings, so it is sometimes referred to as a *moving iron motor*. Rather than being in the gap as with the moving coil motor, the coils are wound in slots cut into the rotor. One obvious advantage of this structure is that the air gap can now be made as small as manufacturing tolerances, thermal expansion, and centrifugal force will allow.



One apparent *disadvantage* is that it seems that it should not work. Because of the high permeability of the iron of the rotor, nearly all of the flux is diverted around the winding and the **B** field at the conductor is essentially zero.



The fact that millions of motors like this are sold every year suggests that we might look a bit closer. The previous picture is somewhat misleading, as it shows the **B** field in the vicinity of the conductor *when no current is flowing*. If we look at the *total* field due to both the permanent magnet and the coil when current is flowing we get a slightly different picture.



In this case, the field is no longer symmetric; it is more intense at the upper edge of the slot. If we use the same argument we used with the sheared plate electrostatic actuator, i.e. that the force produced by the field works in the direction which would shorten the field lines, then there would be a net downward force on the edge of the slot, Combined with the corresponding upward force on the opposite side of the rotor, this would produce a counterclockwise torque. Note that this torque is in the same direction as the torque which would be produced directly on the coil if the iron rotor were absent.

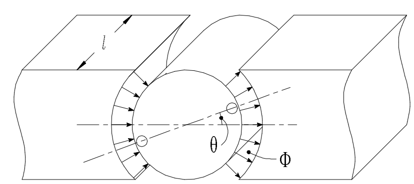
We can use an energy method to quantify the torque. Assuming the energy conversion process is lossless, $dW_e = dW_f + dW_m$. In this case, the field energy is constant, so $dW_f = 0$ and

$$\begin{aligned} P_{in(elec)} &= P_{out(mech)} \\ ei &= T\omega \\ i\frac{d\lambda}{dt} &= T\frac{d\theta}{dt} \end{aligned}$$

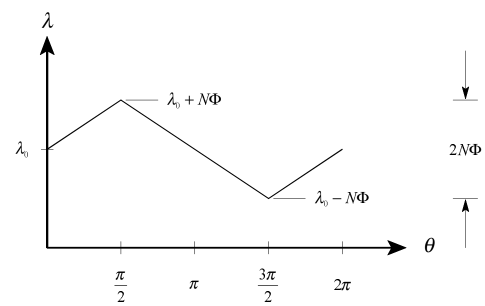
So we have that

$$T = i\frac{d\lambda}{d\theta}$$

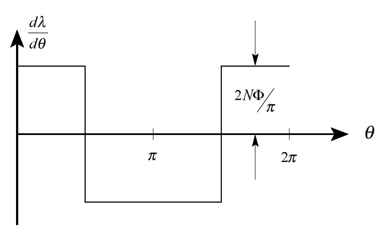
For simplicity, assume the armature contains a single coil of N turns. If the total flux in the gap is Φ, then the portion of that flux linking the coil depends on the angle of the coil. If the gap flux is uniform, this will vary linearly from zero when $\theta = 0$ to $N\Phi$ with the coil in a vertical plane.



To this we must add the flux linkage λ_0 due to the current in the coil, which doesn't vary with angle.



Differentiating with respect to θ gives a torque which is constant in magnitude and changes sign when the coil passes through the vertical plane.



The total flux is $\Phi = BA = B\pi rl$ so

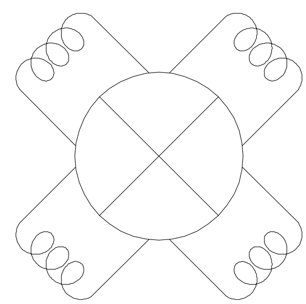
$$T = i\frac{d\lambda}{d\theta} = i\frac{2NB\pi rl}{\pi} = 2NBilr$$

which is the same as for the moving coil motor (8.1).

8.4 Back EMF and the Tachogenerator

The equations we have developed so far do not take into consideration the rotational velocity of the motor. If we force a constant current through the armature, we get a constant torque. However, as with the voice coil, when the conductors in the rotor move through the magnetic field of the stator, an electromotive force or voltage is generated.

Each segment of length of length l in the gap produces a voltage $E = Blu$ where $u = \omega r$ is the tangential velocity of the conductor. Each turn of the coil has two segments of length l in the gap, so each contributes a voltage of $2Blr\omega$.



The total of N turns in the windings appears to the brushes as two coils of $N/2$ turns in parallel. Thus the total voltage at the brushes is

$$E = NBlr\omega = K_e\omega$$

Where $K_e = NBlr$ is called the *back emf constant* of the motor, and has the same value as K_t.

The fact that $K_e = K_t$ is due to the consistent units we have been using and requires that when we state that $T = K_t I_a$ or $E = K_e\omega$ we measure torque in newton-meters and angular velocity in radians/second. Most product data sheets for real world motors will use different units such as inch-ounces and rpm, in which case the values given for the torque and back emf constants will **not** be equal. However, given one of these constants in any set of units, it is always possible to find the other by converting it to the units we have been using.

The fact that a voltage is produced across the motor terminals should not be surprising. For one thing it is necessary so that the electrical input power ($P_e = I_a E$) matches the mechanical output power ($P_m = T\omega$). Also, we saw the same behavior with the voice coil, where a loudspeaker and dynamic microphone are essentially the same structure.

The DC motor is a reciprocal energy conversion device: we can put electrical power in and mechanical power out, using it as a motor, or we can mechanically turn the shaft to convert mechanical power to electrical power in which case it becomes a DC generator. If we are interested in information, rather than power, we can use just the voltage produced, $E = K_e\omega$, without drawing significant current. This gives us a rotational velocity sensor where the output (voltage) is linearly proportional to the velocity. When used for this purpose it is called a *tachogenerator*.

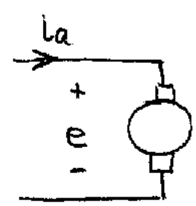
8.5 Permanent Magnet DC Motors

In the motors we have seen so far, the magnetic field in the gap has been produced by one or more permanent magnets. This produces what in many ways is an ideal motor. They are small and require no inputs other than the driving input. The linear relationships between torque and current and between steady state speed and voltage make them easy to use in control systems. In small sizes DC motors are very inexpensive, but at higher power levels, the increasing amount of permanent magnet material rapidly raises the cost.

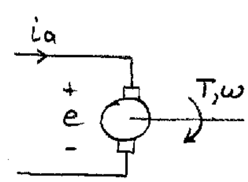
8.5.1 PM DC Motor Characteristics

Up to now, we have implicitly been driving the DC motor with a current source, producing an output torque linearly proportional to the input current. However, the in a voltage oriented world, we need to know what happens when we connect a motor to a voltage source.

First we need a schematic diagram symbol for the DC motor. This consists of a stylized representation of the commutator and brushes.

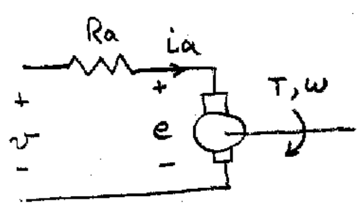


However, this is only half of the picture, because the motor has mechanical as well as electrical terminals. We must consider these, along with the coupling equations, $T = K_t i_a$ and $e = K_e \omega$, whenever we use a motor in a circuit.



Connecting this to a voltage source seems a bad idea: in the absence of any current limiting mechanism, a huge current would flow, producing a huge torque that would quickly accelerate the load to dangerous speeds.

Fortunately, two things conspire to prevent this. First of all, any real motor will have some non zero resistance R_a in the armature winding. This will at least limit the armature current to a finite value. More significantly, as the angular velocity of the motor (and its load) increases, so does the back emf. At some point a speed will be reached where the back emf balances the applied voltage and no further acceleration occurs. If the only load torque is inertial, i.e. if there is no frictional or torsional load, then in the steady state no torque will be required to sustain the speed. In this case, $i_a = T/K_t = 0$, $e = v$ and $\omega = v/K_e$. In other words, the steady state speed is proportional to the applied voltage.



When we combine the effects of the armature resistance and the back emf, we get the following relationships among voltage, current, torque, and velocity.s Using KVL

$$v = R_a i_a + K_e \omega$$

From Ohm's law

$$i_a = (v - e)/R_a = (v - K_e \omega)/R_a$$

Then the torque is

$$T = K_t i_a = \frac{K_t}{R_a}(v - K_e \omega) = K_t \frac{v}{R_a} - \frac{K_t K_e}{R_a}\omega \tag{8.2}$$

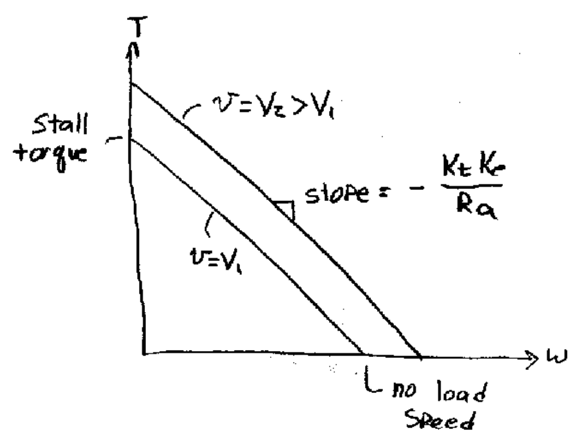
v/R_a is the current which would flow if $\omega = 0$, i.e. if the motor were *stalled*, and is called the *stall current*. The torque produced with the motor stalled, $T = K_t \frac{v}{R_a}$, is called the *stall torque*.

If we solve 8.2 for ω we get

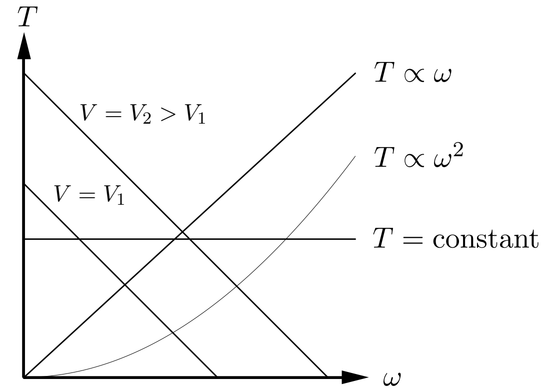
$$\begin{aligned} R_a T &= K_t(v - K_e \omega) \\ \frac{R_a T}{K_t} &= v - K_e \omega \\ \omega &= \frac{v}{K_e} - \frac{R_a T}{K_t K_e} \end{aligned} \tag{8.3}$$

v/K_e is the speed that would result if the load torque were zero and is called the *no load speed*.

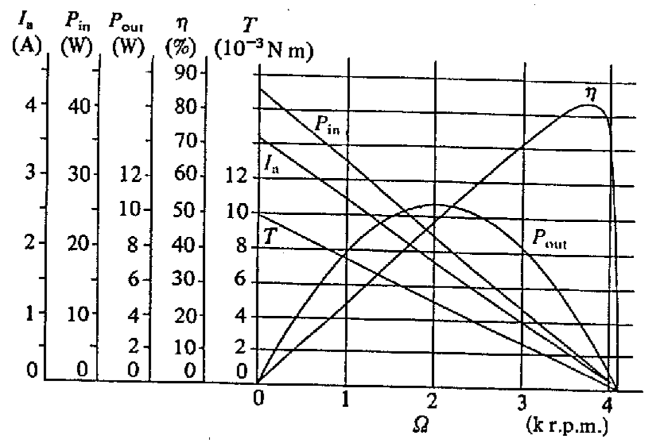
If we plot the torque vs. speed for a fixed applied voltage, we get a straight line which crosses the T axis at the stall torque, and crosses the ω axis at the no load speed. If we change the applied voltage, the curve moves parallel to the original curve.



If we draw these lines for several different voltages, we get a family of curves describing how the speed/torque relationship varies with voltage. If on this same plot we draw the curve representing the speed/torque relationship of the load, then the intersection of this *load line* with one of the curves gives the steady state speed for the corresponding voltage, and the locus of the intersections with curves for different voltages gives the variation of speed with voltage for that load.



Constant Voltage Performance Curves We can also plot a number of other variables as a function of speed for a fixed voltage. Since current is proportional to torque and electrical input power is proportional to current, these both give straight lines like the torque plot. More interesting are the plots of output power and efficiency (η). P_{out}, being the product of T and ω is a parabola with maximum value at one half the no load speed. Although this is the operating point which gives the *maximum* power, it may exceed the *rated* power for the motor. If we want the maximum *efficiency*, important for battery powered equipment, then the motor must at nearly its unloaded speed where unfortunately, the output power is much less than *its* maximum value.



8.6 Wound Field Motors

Up to now the stator field has been provided by a permanent magnet, but it could also be provided by an electromagnet. Figure 8.1 shows a number of ways this might be done, along with a few "equivalent" permanent magnet structures. For obvious reasons, motors using an electromagnet to provide the stator field are called *wound field motors.*

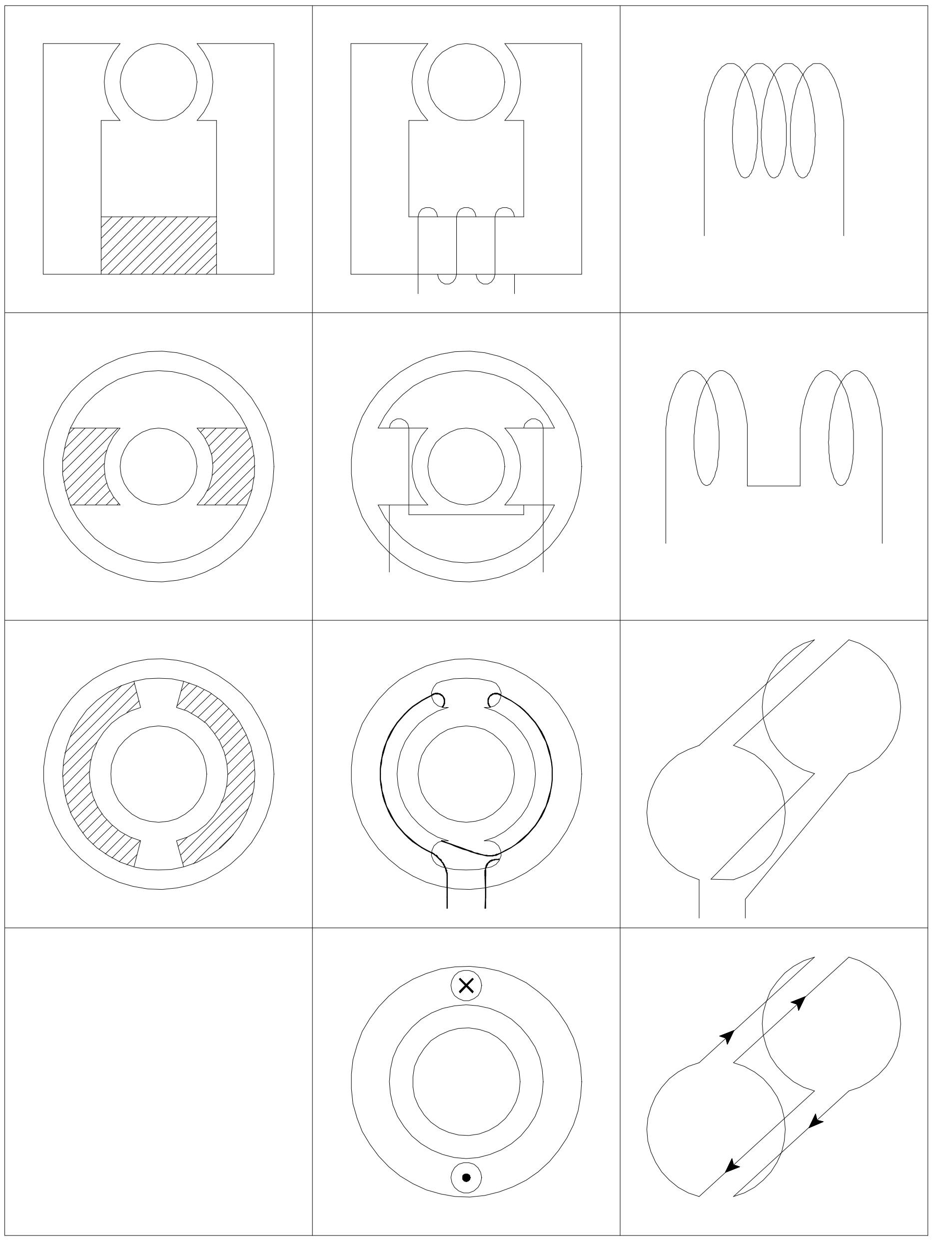
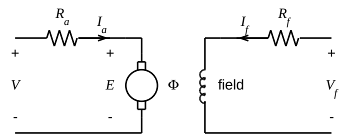


Figure 8.1: Some field configurations for PM and wound field DC motors.

In such a motor, the stator field strength will depend on the value of i_f, the current in the field winding. In terms of the total airgap flux we have $\Phi = \Phi(i_f)$, or for the linear region of the stator material $\Phi = K_f i_f$.

We can modify our circuit diagram to indicate the presence of the field winding and the associated flux.



There are several possible sources for the field current:

1. It could be connected to a source which is independent of the armature supply (separate excitation).
2. It could be connected in parallel with the armature winding (shunt wound).
3. It could be connected in series with the armature winding (series wound).

Each of these gives a motor with different speed/torque characteristics.

For a fixed value of I_f (or V_f) the field flux Φ will be fixed so a separately excited motor with a fixed field supply behaves like a permanent magnet motor.

8.6.1 Shunt Wound Motor

With the field winding connected in parallel with the armature, the field voltage V_f must be equal to the armature voltage V. Substituting $K\Phi$ for K_e in 8.3 we have

$$\omega = \frac{V}{K_e} - \frac{R_a T}{K_t K_e} = \frac{V}{K\Phi} - \frac{R_a T}{(K\Phi)^2}$$

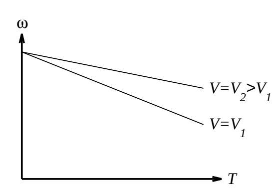
Then the no load speed (ω_{nl}) is

$$\omega_{nl} = \frac{V}{K\Phi} = \frac{V}{KK_f I_f} = \frac{V}{KK_f \frac{V_f}{R_f}}$$

But, since $V_f = V$, we have $\omega_{nl} = \frac{R_f}{KK_f}$ independent of the armature voltage V.

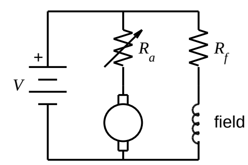
If the load torque is non zero,

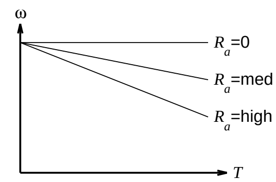
$$\omega = \frac{R_a}{KK_f} - \frac{{R_f}^2 R_a T}{K^2 {K_f}^2 V^2}$$



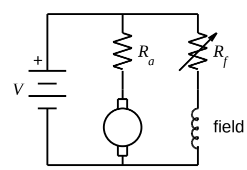
Unlike the family of speed/torque curves for the permanent magnet (or separately excited) DC motor, which is a series of parallel lines, the curves for the shunt wound motor are a set of lines passing through the ω axis at ω_{nl} having slope which varies as the square of the armature voltage.

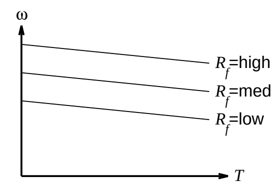
This makes controlling the speed of a shunt wound motor a bit different from controlling a PM motor. On the one hand, if what we want is constant speed with a variable supply voltage, the shunt wound motor provides good speed regulation with a light load. On the other hand, with a larger load we can vary the speed by increasing the effective armature resistance with a series resistor. This changes the intersection of the load line with the speed torque curve by varying the slope of the latter.





To vary the speed with a light load and constant supply voltage we can vary the no load speed by varying the effective *field* resistance with a series resistor.

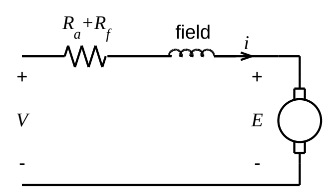




Note that in this case, somewhat unintuitively, the speed *increases* as we increase the field resistance. This suggests the question of what might happen if we let $R_f = \infty$ and the answer is often "bad things." If the field current is removed once the motor is running the speed will increase until limited by the residual magnetism in the field iron, the friction in the bearings, or the strength of the rotor assembly under centrifugal force.

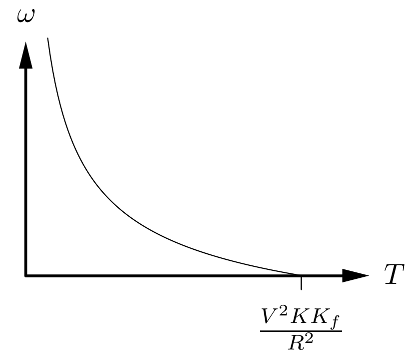
8.6.2 Series Wound Motor

If we connect the field winding in *series* with the armature, then the field *current* i_f will be equal to the armature current i_a, i.e. $i_f = i_a = i$.



If we are in the linear region of the core material, $K\Phi = KK_f i_f = KK_f i$ so that

$$T = K\Phi i = KK_f i^2$$
$$i = \sqrt{\frac{T}{KK_f}}$$



Then the speed is

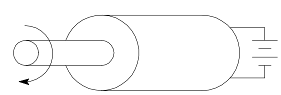
$$\begin{aligned}\omega &= \frac{V}{K\Phi} - \frac{RT}{K^2\Phi^2} \\ &= \frac{V}{KK_f i} - \frac{RKK_f i^2}{K^2{K_f}^2 i^2} \\ &= \frac{V}{\sqrt{KK_f T}} - \frac{R}{KK_f}\end{aligned}$$

where $R = R_a + R_f$.

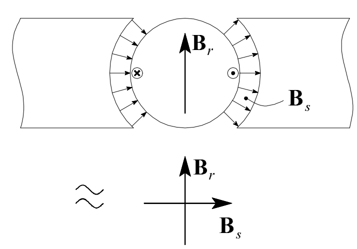
8.7 Brushless Motors

The key element of the DC motor is the commutator, which keeps the current in the rotor windings always in the same direction with respect to the stator field. Another way of looking at the function of the commutator is that it always keeps the field of the rotor at right angles to that of the stator, the condition of maximum torque.

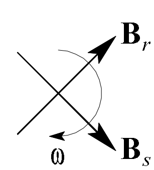
Consider the following experiment: (1) glue a battery to the back of a permanent magnet DC motor, (2) connect the battery and the motor, (3) hold the shaft. We still have a rotary actuator, but now the outer part (containing the permanent magnets) rather than the inner part (containing the coils) is rotating.



Previously when we ran a DC motor, the commutator switched the current in the armature windings so that the field of the rotor remained perpendicular to the field of the stator, resulting in maximum torque as the fields tried to align.

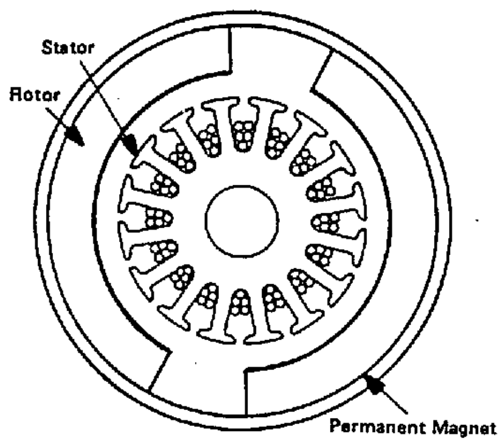


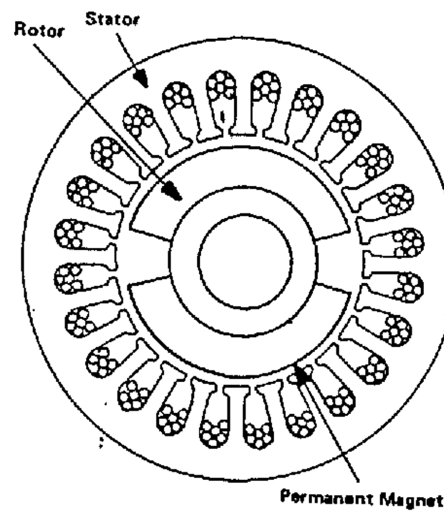
Holding the shaft rather than the frame of the motor doesn't change what the commutator is doing: the two fields are still kept perpendicular, but now they are both rotating. However, the rotor is now the part with the permanent magnets and the stator is the part with the coils.



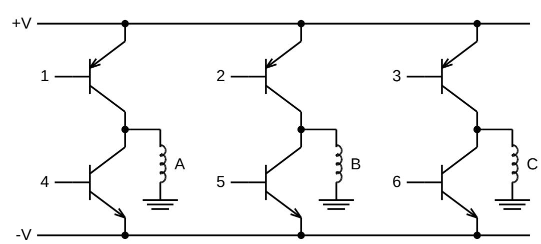
There are two problems with this arangement: (1) in most applications we want the shaft of the motor to rotate, not the frame; (2) having to rotate the power supply along with the load is undesirable.

We can fix problem 1 by rearranging the motor so that the magnets are on the inside and the coils are on the outside. This puts the rotor and stator back in the places we're used to.

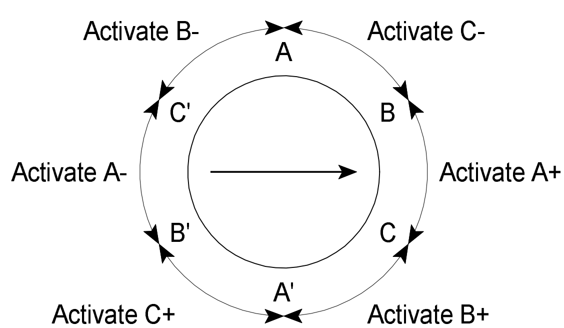




We solve problem 2 by realizing that since the coils are now on the outside and are stationary, we don't need a mechanical commutator to switch the current through them, but instead can use transistors. Here's a circuit that will do this (for a stator winding with three coils):



The trick is to know *when* to switch the currents. Recall that what we want to do is to keep the fields produced by the rotor and stator perpendicular (or nearly perpendicular). We can do this by watching the North pole of the rotor and switching current to the appropriate winding depending on where it is pointing. This is usually done with a *Hall sensor* which we will study later.



8.8 Summary

Here's a device scorecard for the different types of DC motors.

Classification			
Class Name	DC Motor		
Subclass Name	Permanent Magnet	Wound Field	Brushless
Sensor or Actuator	Actuator		
Type of Motion	Rotational		
Intended Output	torque or velocity		
Behavior			
Controlling input	current or voltage		
Defining equations	$T = K_t i_a$ $\omega = \frac{v}{K_e} - \frac{R_a T}{K_t K_e}$	$T = K\Phi i_a$ $\omega = \frac{R_a}{KK_f} - \frac{{R_f}^2 R_a T}{K^2 {K_f}^2 V^2}$ (shunt) $\omega = \frac{V}{\sqrt{KK_f T}} - \frac{Ra}{KK_f}$ (series)	$T = K_t i_a$ $\omega = \frac{v}{K_e} - \frac{R_a T}{K_t K_e}$
Capability			
Range	θ: continuous, ω: 0 to 20k rpm		
Load Rating	< 1 kW	> 100 W	< 1 kW
Interface			
Voltage	< 100 V		< 100 V
Dynamics			
Mass	low (moving coil)		
Physical, Economic			
Size	small		small
Form Factor	cylindrical		
Cost	low	moderate	moderate
Efficiency	good		
Reliability	good		excellent
Maintenance	brushes		none
Environmental			
Safety	sparking		
Miscellaneous			
Integratability	fair		good

Figure 8.2: DC Motor Scorecard