Abstract—A broad class of CSMA protocols use synchronized contention in which nodes periodically contend at intervals of fixed duration. Both analysis and operation of these protocols assume perfect clock synchronization. However, in practice, clocks are not perfectly synchronized and drifts can vary widely, depending on the synchronization mechanism used in the network. We introduce an analytical model that quantifies the interplay of clock drifts with key performance factors such as contention window size, carrier sense, and usage of guard time. Utilizing this model, we investigate the isolated and combined impact of these factors to per-flow throughput and fairness properties in both single-hop and multi-hop networks. Our model reveals conditions on protocol parameters under which the throughput of certain flows can exponentially decrease with clock drift; while at the same time, it enables solutions that can offset such problems in a predictable manner.

I. INTRODUCTION

Synchronized CSMA protocols partition time into periodic cycles of fixed duration. At the beginning of each cycle, nodes contend on a common channel using carrier sense and control packet handshake such as RTS/CTS; after winning contention, nodes transmit until the end of the cycle. Synchronized CSMA contention has been used in both single-hop and multi-hop wireless networks for a wide variety of tasks. In single-hop networks, the IEEE 802.11 WLAN standard [1] supports synchronized contention in power-saving mode such that hosts wake up periodically and contend using beacons in a short window. In multi-hop networks, synchronized contention has been used for channel selection [14] and per-channel contention [2] in multi-channel networks, for leader election and power saving in sensor networks [5], [16], and for antenna selection in MIMO networks [10].

In practice, clock drifts can range from sub-microseconds to several msec, depending on the clock synchronization protocol’s trade off between clock accuracy and cost or complexity (see for example [13], [11]). Drift impacts medium access in that with the combination of synchronized contention and clock drift, some flows may start contention earlier and thus have higher chances to win the channel than others. This paper is the first to systematically and comprehensively study via analytical models and simulations the fairness properties and the impact of key performance factors in synchronized CSMA systems, with a particular focus on clock drift. Our contributions are as follows.

First, we define S-CSMA, a general protocol that captures the basic features of synchronized CSMA contention. We then identify the main factors that affect the fairness properties of S-CSMA, namely imperfect synchronization, control packet size and carrier sense.

Second, we begin our analysis of S-CSMA by studying the impact of imperfect synchronization in single-hop networks (e.g., synchronized WLANs). To this end, we introduce an accurate Markovian model for throughput prediction that incorporates clock phase drifts. Existing Markovian models for asynchronous CSMA protocols such as 802.11 [3], [6], [9], [12], [15] can accurately model stochastic contention instants due to carrier sense, but are not suitable for periodic contention. Our Markovian model is different in nature and exploits the periodic structure of synchronized contention. For the single-hop case, we find that flows with the earliest (latest) clock phases receive maximum (minimum) throughput, and that minimum throughput decreases exponentially with clock phase drift. Surprisingly, both phenomena occur irrespective of the use of guard time.

Third, we consider multi-hop networks which have a complication that in addition to their clock phase differences, flows compete with an asymmetric view of channel state and can suffer from a lack of transmission opportunities due to carrier sensing. We extend our Markovian model to simple multi-hop scenarios that are representative of these phenomena. Unlike the single-hop case, our model predicts that flows with the earliest clocks do not necessarily receive the highest throughput. Instead, they may even receive zero throughput under certain conditions that we identify. For both guard-time and no-guard-time systems, we derive simple relationships among flow contention window sizes and clock phases that guarantee starvation-free operation. According to these relationships, in no-guard-time systems, each flow requires clock phase knowledge of two-hop neighbors; while in guard-time systems, of one-hop neighbors. This implies different requirements on the phase bounds provided by the clock synchronization mechanism running in the network.

Finally, we consider arbitrary topologies and introduce an approximate model that lower bounds throughput for guard-time systems. Our approximation allows us to accurately decouple the joint effect of interfering flows and explicitly express the throughput of each flow as a function of its one-hop neighborhood. More specifically, the throughput of each flow depends on its own contention window size and the harmonic mean of the contention window sizes of its one-hop interferers. In addition, if a flow has a topology and/or clock phase disadvantage, its throughput will decrease exponentially with respect to control packet size and/or the average relative phase of its one-hop interferers, respectively. Our model reveals that this disadvantage cannot be offset by making this flow more aggressive (i.e., by decreasing its contention window), but only by making its interfering flows less aggressive (namely, by increasing the harmonic
mean of their contention windows). Using our approximate model for arbitrary topologies, we study the impact of fairness objectives on contention window sizes. We find that objectives such as max-min fairness or proportional fairness can require large contention window sizes for all flows, even in simple topologies: larger contention window sizes require larger cycle durations to absorb the contention window overhead, which can lead to increased network delays.

The remainder of this paper is organized as follows. In Section II, we introduce S-CSMA and identify the key performance factors that affect fairness. In Section III, we introduce the Markovian model to study imperfect synchronization in single-hop wireless networks. We generalize this model to representative multihop scenarios in Section IV. In Section V we model arbitrary topologies for guard-time systems, investigate the impact of interference on the resulting throughput approximation and the impact of fairness objectives on contention window adjustment mechanisms. Section VI concludes.

II. S-CSMA PROTOCOL

The detailed operations of synchronized CSMA protocols are all different from each other and, as a result, there is no single protocol model that characterizes all the operational details of these protocols. To analyze the basic nature of synchronized contention and capture its fundamental fairness properties, in this section we present a simple Synchronized CSMA protocol, which we call S-CSMA. Our protocol is not designed to capture all the details of this protocol family nor for improving performance. It is designed to be simple but characterize the basic use of synchronized contention. The principle and methodology of our analysis can be extended to the analysis of a particular protocol.

As shown in Fig. 1, S-CSMA is a single-channel synchronized CSMA protocol where time is partitioned into fixed-duration cycles. Each cycle consists of a contention phase followed by a data transmission phase. At the beginning of each cycle, each node $i$ starts contending for the channel by sensing the medium. Once the medium becomes idle, the node computes a random backoff counter based on an initial contention window $W_i$ and starts counting down mini-slots while sensing the medium. If during this time a transmission is sensed, the node quits contention and waits for the next cycle. If the backoff timer expires, the node transmits a short request (REQ) control packet and waits for a short grant (GNT) control packet. If the GNT is received, the data phase begins immediately and the node transmits data until the end of the cycle. The data transmission phase may consist of one or more data-link frames and their corresponding acknowledgments. If the node does not receive a grant packet, it starts counting down its contention window and continues to send data until the end of the cycle. The degree of topological bias is essentially determined by the duration of the REQ control packet.

A system design alternative is to employ a guard time at the end of the data phase. In such a guard-time system, upon finishing data transmission, each flow waits for an additional duration $T_g$ before starting contention for the next cycle. A guard-time system may avoid starvation problems due to carrier sense, but on the other hand, it takes away potential opportunities to alleviate the phase bias.

Phase bias, topological bias, carrier sense and the use of guard time are tightly coupled and can dramatically affect the fairness properties of multi-hop synchronized CSMA systems. In the next sections, we introduce models that treat such systems in a unified manner and address critical design issues such as (i) impact of phase bias and topological bias to per-flow throughput (ii) effectiveness of carrier sense in alleviating phase bias (iii) requirements for starvation-free clock synchronization mechanisms (iv) guidelines for contention window adjustment mechanisms to address unfairness due to both phase and topological bias.

III. MODELING SINGLE HOP NETWORKS

In this section, we develop a discrete time Markov chain model to predict per-flow throughput in a single-hop network that employs S-CSMA. We account for temporal (dis)advantages introduced by different clock phases in our model, where the system state for a given cycle represents which flow transmits during this cycle. The transition probability is determined by relative clock phases of the flows and other system factors. This allows us to relate clock phase to the stationary distribution of the system state. Using this

Since the node clocks are not perfectly synchronized, leading flows whose transmitters begin contention earlier at each cycle have a phase advantage over lagging flows. Carrier sensing is known to play a fundamental role in affecting the fairness properties of asynchronous CSMA protocols such as IEEE 802.11[6]. In synchronized protocols, it is also an important factor that may either significantly alleviate or aggravate the phase bias introduced by clock drifts.

In addition to phase bias, a multi-hop S-CSMA network exhibits topological bias. More specifically, a flow $A$ has a topological advantage with respect to flow $B$ if the transmitter of $A$ is within range of the receiver of $B$ but not its transmitter. Being a hidden terminal, the advantaged flow can be blocked only when its transmitter senses the GNT packet of the disadvantaged flow receiver. Thus, the disadvantaged flow can win contention only if it completes random backoff countdown plus REQ packet transmission before the advantaged flow completes its random backoff count-down.

The degree of topological bias is essentially determined by the duration of the REQ control packet.

Fig. 1. Basic operations of S-CSMA


drawings
analytical model, we investigate the joint effects of imperfect synchronization, carrier sense and guard time.

Our throughput metric is per-flow success probability, defined as the probability that a flow successfully reserves the channel at a cycle. In the remainder of the paper, we use success probability and throughput interchangeably, both referring to the same metric. We also use the terms contention window, backoff counter, and phase for flows instead of their transmitter nodes. Thus, these quantities of a flow are the corresponding quantities of its transmitter.

A. Analytical Model

Let \( t_i(k) \) be the time instant where the \( k \)-th cycle of node \( i \) begins. Let \( T_c \) and \( T_d \) denote the duration of the contention phase and the data transmission phase, respectively. The fixed duration of a cycle is then given by \( T = T_c + T_d + T_{gb} \), where \( T_{gb} \) is the duration of guard time. We characterize each node by its clock phase \( \theta_i \), with respect to an absolute global clock reference (or alternatively, the earliest clock in the network). Given the phase \( \theta_i \), the contention time instant of each node \( i \) for cycle \( k \) is given by:

\[
t_i(k) = \theta_i + kT, \quad k = 0, 1, 2, \ldots
\]  

We assume that the clock frequencies of all flows are equal and constant and that the clock phase difference between any two flows can be at most \( \theta_{\text{max}} \). \( \theta_{\text{max}} \) can be either known (as a maximum error of a clock synchronization mechanism) or unknown to the flows. We assume \( \theta_{\text{max}} \) is much smaller than \( T \), the duration of a cycle, and is smaller than \( T_{gb} \) in a guard-time system. We also denote by \( \theta_{ij} = \theta_i - \theta_j \) the relative phase between nodes \( i \) and \( j \). All these quantities are expressed in mini-slots.

**System state:** Consider a fixed number \( N \) of contending flows in the network, indexed by \( 1, 2, \ldots, N \), respectively. We let \( b(k) \) denote the index of the flow accessing the channel at cycle \( k \). We model the evolution of stochastic process \( b(k) \) by a discrete time Markov chain, in which the state of the system in cycle \( k \) is \( b(k) \), and the state space is \( S = \{1, 2, \ldots, N\} \).

The transition probability from state \( i \) to state \( j \), denoted by \( p_{ij} = P\{b(k) = j | b(k-1) = i \} \), is the probability flow \( j \) wins contention at cycle \( k \) given that flow \( i \) transmitted during cycle \( k - 1 \). Note that the transition probabilities do not depend on \( k \) because in a synchronized CSMA system the flows refresh their contention state at the beginning of each cycle. Solving the Markov chain, we obtain the stationary distribution \( \pi = \{\pi_i\}, \forall i \in S \), where \( \pi_i \) is the success probability of flow \( i \).

**Transition probability:** We now compute the transition probabilities \( p_{ij} \) for both guard-time and no-guard-time systems. We denote by \( X_i \) the random backoff counter computed by each flow \( i \) at a contention cycle and use \( p_i(x) \) to denote \( P(X_i = x) \) and \( \Phi_i(x) \) to denote \( P(X_i > x) \).

**No-guard-time system:** Let \( i \) be the flow transmitting during cycle \( k - 1 \). We divide all other flows into two sets, the leading set \( LD(i) = \{m: \theta_m \leq \theta_i\} \) and the lagging set \( LG(i) = \{m: \theta_m > \theta_i\} \). At the beginning of cycle \( k \), flow \( i \) will sense idle medium and begin random backoff at its contention instant \( t_i(k) \). Any flow \( m \) in leading set \( LD(i) \) will also begin backoff at \( t_i(k) \) because it will sense data transmissions of flow \( i \) during cycle \( k - 1 \). On the other hand, any flow \( m \) in lagging set \( LG(i) \) will start backoff at its later contention instant \( t_m(k) \), provided neither flow \( i \) nor its leading flows have counted down their backoff counters to zero by that time instant.

Now, let \( j \) be the flow that wins contention at cycle \( k \). If flow \( j \) is in set \( \{i\} \cup LD(i) \), and its backoff counter \( X_j = x \), it will win contention if (i) all other nodes in \( \{i\} \cup LD(i) \) use a random backoff counter greater than \( x \); (ii) the random backoff counters of all nodes in lagging set \( LG(i) \) expire after \( t_i(k) + x \). On the other hand, if flow \( j \) is in the lagging set \( LG(i) \), it will win contention only if the random backoff counters of all other nodes expire after \( t_j(k) + x \). After taking expectation with respect to \( x \), the transition probability \( p_{ij} \) is given by,

\[
p_{ij} = \begin{cases} 
\sum_{x=0}^{W_i-1} p_j(x) \prod_{m: \theta_m \leq \theta_i, m \neq j} \Phi_m(x) & \theta_{ji} \leq 0 \\
\sum_{x=0}^{W_i-1} p_j(x) \prod_{m: \theta_m > \theta_i} \Phi_m(\theta_{jm} + x), & \theta_{ji} > 0 
\end{cases}
\]  

Eq. (2) does not handle collisions, resulting in the sum of the transition probabilities out of a system state being slightly less than 1 (\( \sum_j p_{ij} < 1 \)). To handle collisions, we add an additional collision state \( c \) to the system state space. The transition probability from state \( i \) to state \( c \) is \( 1 - \sum_j p_{ij} \). To compute \( p_{jc} \), we assume the time it takes flows to detect and handle collision at the beginning of the cycle is larger than the clock phase differences. Since clock phases are absorbed, we assume each flow has equal access opportunity after a collision, i.e. \( p_{jc} \) is equal for all \( j \). Extensive simulations have indicated that this simplifying assumption does not compromise the model’s accuracy.

**Guard-time system:** When guard time is present, no node will sense a busy channel at its contention instant of cycle \( k \) and will immediately start random backoff. Since carrier sense at the beginning of each cycle has no effect, the transition probabilities \( p_{ij} \) are independent of which node \( i \) transmitted during the previous cycle. Therefore, the transition probabilities \( p_{ij} \) for the guard time system are given by:

\[
p_{ij} = \sum_{x=0}^{W_j-1} p_j(x) \prod_{m: m \neq j} \Phi_m(\theta_{jm} + x).
\]  

Collisions are handled by adding a collision state, similar to the no-guard-time system. Eqs. (2) and (3) allow us to relate factors such as the relative clock phases to the transition probabilities. We next investigate the impact of these factors on per-flow success probability using our analytical model.

B. Clock drift investigation

**Fixed clock phases:** We consider 10 flows in a single-hop network. Each flow \( i \) uses a contention window of \( W_i = 32 \) mini-slots and draws its backoff counter uniformly within this window in each contention cycle. The clock phases are fixed to \( \theta_i = i \times 10, i = 0, \ldots, 9 \) mini-slots, yielding a maximum relative phase equal to 90 mini-slots.

We first consider a no-guard-time system. Fig. 2(a) depicts the success probabilities \( \pi_j \) predicted by our model and
throughputs obtained by \( ns \) simulations as a function of the flow clock phases sorted in non-decreasing order. We observe an excellent match and that throughput falls off sharply with the phase distance from the earliest clock, which can be explained by our model: Eq. (2) predicts that when the clock phase of flow \( j \) becomes larger, \( p_{ij} \) becomes smaller, and at the same time \( p_{ji} \) becomes larger for all \( i \), thus rapidly decreasing the stationary probability of state \( j \). The guard-time system, also depicted in Fig. 2(a), follows a similar trend as the no-guard-time system. Thus, different clock phases can lead to unfairness or even starvation, regardless of whether guard time is used or not.

Random clock phases: We now model per-flow throughput when the phases are random variables following a common truncated normal distribution of maximum phase \( \theta_{\text{max}} \). This is equivalent to studying per-flow throughput, averaged over several simulation runs, where at each run the phases are picked according to this distribution and remain fixed during the run. Fig. 2(b) depicts the evolution of the lowest throughput in a guard-time system as a function of \( \alpha = \theta_{\text{max}}/W \). The main observation is that the lowest throughput decreases exponentially with \( \alpha \). A similar trend also holds for the no-guard-time system, underlining the severe effect of clock drift on phase-disadvantaged flows even when carrier sense is enabled.

IV. ANALYZING THE ROLE OF CARRIER SENSE IN S-CSMA MULTIHOP NETWORKS

In CSMA wireless networks, carrier sense can yield unfairness or even starvation when flows sense uncoordinated transmissions in their neighborhood. A representative scenario is Flow-in-the-Middle (FIM), shown in Fig. 3. In asynchronous CSMA protocols like 802.11, the middle flow receives very low throughput [4]. Since the outer flows are not within sensing range of each other, their transmissions are not time-aligned. As a result, the middle flow can only contend in the small intervals where both outer flows are jointly idle, which results in very few transmission opportunities. In contrast, under S-CSMA the middle flow throughput can vary from zero to maximum depending on the relative clock phases of the flows and their interaction with carrier sense. By extending the model of Section III, we use the FIM scenario to analyze these interactions and also evaluate the option of disabling carrier sense using guard time.

A. Analytical model

The key observation that simplifies the analysis of the FIM scenario is that when one of the outer flows wins contention in a cycle, the other outer flow will also transmit in the same cycle, while the middle flow will not transmit. On the other hand, when the middle flow wins, both outer flows defer transmission during this cycle. We use two states in the model: state 1 for the two outer flows and state 2 for the middle flow.

Without loss of generality we assume that flow 1 is the earlier outer flow (\( \theta_1 \leq \theta_3 \)). We compute the transition probabilities \( p_{12} \) and \( p_{22} \) of middle flow 2 winning contention during cycle \( k \), given that either the outer flows or the middle flow itself were transmitting during cycle \( k-1 \), respectively. Then the other two transition probabilities can be determined as: \( p_{11} = 1 - p_{12} \) and \( p_{21} = 1 - p_{22} \). Given the transition probabilities, the success probability of the middle flow is given by the stationary probability closed form expression of a two-state Markov chain:

\[
\pi_2 = \frac{p_{12}}{1 + p_{12} - p_{22}}
\]

while the success probability of each of the outer flows is \( \pi_1 = 1 - \pi_2 \).

Transition probability of no-guard-time system. The transition probabilities depend on how the clock phase of the middle flow is related to the phases of the outer flows. We consider the case where the middle flow 2 is leading both outer flows, i.e., \( \theta_2 \leq \theta_1 \leq \theta_3 \). To compute \( p_{12} \), the transition probability of the middle flow winning the contention in cycle \( k \) given the two outer flows transmitted in cycle \( k-1 \), we need to determine the time instants when the three flows start their backoff counters. As shown in Fig. 4, the two outer flows sense an idle channel at their contention instants for cycle \( k \) and start their backoff counters immediately. However, due to carrier sense, flow 2 starts its backoff counter only when flow 3 finished its transmission for cycle \( k-1 \). Once the time instant for each flow to start its backoff counter is determined, the transition probability can be computed accordingly. The transition probabilities in different cases are summarized in Table 1.

![Fig. 4. Computation of \( p_{12} \), the probability of middle flow 2 winning contention given the two outer flows transmitted in previous cycle. Arrows represent cycle boundaries.](image-url)
Transition probability of guard-time system. When guard time is used, no flow senses busy carrier at the beginning of each cycle. Thus, in this case, the transition probabilities are computed similarly to the single-hop guard time case:

\[ p_{i2} = \sum_{x=0}^{W_2-1} p_2(x) \Phi_1(x + \theta_21) \Phi_3(x + \theta_23), \quad i = 1, 2 \quad (5) \]

**B. Clock phase investigation**

The preceding analysis reveals that the success probabilities depend on the relative phases of the three flows and whether guard time is used or not. In this section, we quantitatively study the effect of these factors on the flow success probabilities. Unless otherwise specified, we assume all flows use equal contention windows \( W_1 = W_2 = W_3 = 32 \) mini-slots.

**Effect of relative phase of the outer flows \( \theta_31 \).** We set \( \theta_2 = \theta_1 = 0 \) and vary the phase \( \theta_3 \) of the late outer flow 3 from 0 to 40 mini-slots, as shown in Fig. (5). The performance of the system without guard time is depicted in Fig. 6(a). We observe that the success probability of the model perfectly matches the success probability obtained by ns simulations. As the phase \( \theta_3 \) of the late outer flow 3 increases, the success probability of the middle flow decreases. At \( \theta_3 > 32 \) mini-slots, the middle flow receives zero throughput. In this case, flow 1 starts backoff immediately but flow 2 carrier senses the transmission of flow 3 and delays its contention for an amount of time greater than the contention window of flow 1.

**Effect of middle flow phase \( \theta_2 \).** We now investigate the effect of phase \( \theta_2 \) of the middle flow for a fixed relative phase \( \theta_31 \) of the outer flows. More specifically, contention windows for all flows are set to 32 mini-slots and \( \theta_3 = 0, \theta_3 = 16 \) mini-slots so that the middle flow starvation condition \( (W_1 < \theta_31) \) does not hold. The phase of the middle flow \( \theta_2 \) varies from -40 to 40 mini-slots.

**No-guard-time system:** The success probabilities are depicted in Fig. 7(a). We observe that when the middle flow 2 is leading both outer flows by more than 32 mini-slots, flow 2 receives maximum throughput while the outer flows receive zero throughput. This is because if the middle flow 2 transmits during cycle \( k - 1 \), its backoff counter at cycle \( k \) will always exceed the contention window of flow 1.

| \( \theta_2 \leq \theta_1 \) | \( p_{12} = \sum_{x=0}^{W_2-1} p_2(x) \Phi_1(x + \theta_21) \Phi_3(x + \theta_23) \) |
| \( \theta_1 \leq \theta_2 \leq \theta_3 \) | \( p_{12} = \sum_{x=0}^{W_2-1} p_2(x) \Phi_1(x + \theta_21) \Phi_3(x + \theta_23) \) |
| \( \theta_2 \geq \theta_3 \) | \( p_{12} = \sum_{x=0}^{W_2-1} p_2(x) \Phi_1(x + \theta_21) \Phi_3(x + \theta_23) \) |

**TABLE I**

**Model for FIM scenario in no guard time system: Transition probabilities into state 2.**

---

<table>
<thead>
<tr>
<th>Clock phase (in mini-slots)</th>
<th>Success probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early outer flow 1 (( \theta_3 ) range)</td>
<td>0.5</td>
</tr>
<tr>
<td>Middle flow 2 (( \theta_3 ) range)</td>
<td>1.5</td>
</tr>
<tr>
<td>Late outer flow 3 (( \theta_3 ) range)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

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**Fig. 5.** Clock phase investigation: \( \theta_2 = \theta_1 = 0, 0 \leq \theta_3 \leq 40 \), all in mini-slots.

**Fig. 6.** FIM scenario, flow success probability as a function of \( \theta_3 \).

**Fig. 7.** FIM scenario, flow success probability as a function of \( \theta_2 \).
expire earlier than the time instants when the outer flows start backoff. Hence, once the middle flow 2 wins contention at a cycle, it will continue to do so at every subsequent cycle.

This is confirmed by Table I, where in this case, 

does not fall off as sharply as in regions other than the flat region of the no guard time system.

V. Modeling Arbitrary Topologies

Due to spatial reuse in a multi-hop network, several flows can transmit simultaneously during a cycle. Thus, our Markovian model can be naturally extended to the multi-hop case using one state for each such non-interfering set of flows.

Then the success probability of each flow can be found by adding the stationary probabilities of the non-interfering flow sets it belongs. However, we do not pursue this approach due to (i) complexity of enumerating the independent sets and computing transition probabilities under imperfect synchronization and (ii) accurate prediction of per-flow success probability requires global information.

Instead, we introduce a model that computes a lower bound on the success probability based only on one-hop information. This approach not only greatly simplifies analysis but also enables to directly study the interplay of the factors that affect fairness properties and ultimately enable distributed algorithms that address unfairness by providing minimum throughput guarantees.

The key observation that enables development of such a model in a synchronized CSMA system is the fact that, at the beginning of each cycle, all flows (i) reset their contention windows to minimum and (ii) compute their backoff counters independently. This allows expressing the one-hop neighborhood interference of each flow in product form. This interference expression is a lower bound to the success probability of a flow, because in reality its one-hop interferers can themselves experience interference by the two-hop neighbors of this flow. The second critical step that reduces the model complexity and leads to a closed form expression is a classification of one-hop interferers according to their topological (dis)advantages.

A. Modeling success probability lower bound.

We use a simple geometric interference model characterized by the transmission range $R_T$ and sensing range $R_S$. The transmission range $R_T$ is defined as the maximum distance that allows correct decoding of a packet. The sensing range $R_S$ ($R_S \geq R_T$) is defined as the maximum distance that triggers carrier sensing as well as the maximum distance that can cause collision due to simultaneous reception of more than one transmission. We define the one-hop interfering set $L_i$ of flow $i$ as the set of flows whose transmitter or receiver is within sensing range $R_S$ of either the transmitter or receiver of flow $i$.

We now consider a flow $i$ in isolation and derive a lower bound $b_i$ on its success probability $\pi_i$ as a function of the flows in $L_i$. We divide the one-hop interfering set $L_i$ of flow $i$ in three subsets classified by geometric (dis)advantage with respect to flow $i$:

$R_i$: set of equivalent neighbor flows of flow $i$: Any flow $j$ in this set would have equal success probability with flow $i$ had they been contending in isolation using perfectly synchronized S-CSMA and equal contention windows. This set includes any flow $j$ in $L_i$ that has (i) a common transmitter or receiver node with flow $i$, (ii) a transmitter within sensing range of the flow $i$ transmitter, (iii) a transmitter not in sensing range of the flow $i$ transmitter but a receiver within sensing range of the
flow \( i \) receiver, or (iv) a transmitter only within range of the flow \( i \) receiver and a receiver only within range of the flow \( i \) transmitter. We note that set \( F \) is symmetric, i.e., for any flow \( j \in F_i \) it also holds that \( i \in F_j \).

**A. set of advantaged neighbor flows of flow \( i \):** Any flow \( j \) in this set would have higher success probability than flow \( i \) due to the REQ packet duration, had they been contending in isolation using perfectly synchronized S-CSMA and equal contention windows. For example, in the IA scenario, flow 2 is advantaged with respect to flow 1. More formally, this set includes all flows \( j \in L_i \) for which neither transmitter nor receiver are within sensing range of the transmitter of flow \( i \) and whose transmitter is within range the receiver of flow \( i \).

**D. set of disadvantaged neighbor flows of flow \( i \):** For any flow \( j \) in this set, flow \( i \) is in \( A_j \).

To compute \( b_i \) we assume flow \( i \) only contends with its one-hop neighbors. This is a pessimistic assumption for the success probability of flow \( i \), because its one-hop neighbors can themselves experience interference from the two-hop neighbors of flow \( i \) (namely flows in \( L_j \) that are not in \( L_i \)). This can only increase the success probability of flow \( i \).

At the beginning of each cycle, the backoff counters computed by different flows are independent with each other. Let the backoff counter of flow \( i \) at the beginning of a cycle be \( x_i \) mini-slots. With respect to the set \( F_i \), flow \( i \) will win the contention only if all the equivalent flows’ back-off counters are greater than \( x_i \). With respect to \( A_i \), flow \( i \) will win contention only if all the disadvantaged flows’ back-off counters are greater than \( x_i \) plus REQ packet duration. Finally, with respect to \( D_i \), flow \( i \) will win contention only if all the disadvantaged flows’ back-off counters plus REQ packet duration are greater than \( x_i \). Removing the condition on \( X_i = x_i \) and incorporating relative phases, we reach the following expression for \( b_i \):

\[
b_i = \sum_{x_i = 0}^{W_i - 1} p_i(x_i) \prod_{f \in F_i} \Phi_f(x_i + \theta_{if}) \times \prod_{a \in A_i} \Phi_a(x_i + R + \theta_{ia}) \times \prod_{d \in D_i} \Phi_d(x_i - R + \theta_{id})
\]

(6)

Derived under a pessimistic assumption, \( b_i \) is a lower bound from ns simulations using the parameters of Table 1. Our observations are summarized by a representative experiment for 15 flows operating in a 1000m x 1000m area, shown in Fig. 9. Fig. 9 plots the per-flow throughputs and lower bounds sorted in decreasing order of the throughputs. We observe that the lower bound curve is below and follows the decreasing trend of the throughput curve. In addition, the lower bound becomes tighter for the lower throughput flows. Thus, Eq. (6) reflects congestion conditions in the network while being a good approximation of actual throughput in highly congested regions.

**B. Impact of REQ packet duration**

Here we derive an approximate closed form of Eq. (6) to investigate the impact of REQ packet size on the success probability lower bound. Starting from Eq. (6), we first set all phase terms to zero to isolate the effect of REQ packet duration. We then consider an approximate backoff model where each backoff counter \( X_i \) is geometrically distributed with parameter \( q_i \), instead of being uniformly distributed within \( 0,1,2,\ldots,W_i - 1 \). This approximation has also been successfully used in asynchronous CSMA protocol models [7], [8], [12]. In the geometric backoff model, \( q_i \) is the probability that flow \( i \) will transmit in the next mini-slot if its current window is \( W_i \). As in [12], we set \( q_i = \frac{2}{W_i} \) to achieve the same average backoff counter value with the uniform distribution model. After the above steps, we further approximate Eq. (6) by its continuous form:

\[
b_i = \int_{x_i = 0}^{\infty} p_i^c(x_i) \prod_{f \in F_i} \Phi_f^c(x_i) \times \prod_{a \in A_i} \Phi_a^c(x_i + R) \times \prod_{d \in D_i} \Phi_d^c(x_i - R) dx_i,
\]

(7)

where \( p_i^c() \), \( \Phi_f^c() \) are the continuous counterparts of \( p_i() \), \( \Phi_f() \), respectively.

Since \( X_i \) in discrete form is geometrically distributed with parameter \( q_i = 2/W_i \), in continuous form it is exponentially distributed with parameter \( \lambda_i = 2/W_i \). Substituting \( p_i^c() \) and \( \Phi_f^c() \) corresponding to this distribution in Eq. (7) and after algebraic manipulations, we reach the following closed form expression for the lower bound of flow \( i \):

\[
b_i = \frac{\lambda_i e^{-2RC_a} e^{-R(C_f + \lambda_i)}}{C_f + C_a + C_d + \lambda_i} + \frac{\lambda_i e^{-RC_a}}{C_f + C_a + \lambda_i} \left[ 1 - e^{-(C_f + C_a + \lambda_i) R} \right]
\]

(8)

where \( C_f = \sum_{f \in F_i} \lambda_f \), \( C_a = \sum_{a \in A_i} \lambda_a \), and \( C_d = \sum_{d \in D_i} \lambda_d \).

According to Eq. (8), \( b_i \) is jointly determined by the one-hop interferers of flow \( i \) in the set \( L_i \), yet each subset \( F_i \), \( A_i \) and \( D_i \) of \( L_i \) contributes differently. Next, we investigate the contribution of each individual subset.

**Impact of equivalent interfering flows \( F_i \):** The expression for \( b_i \) considering only the flows in \( F_i \) is obtained by setting...
$D_i = 0, A_i = \emptyset$ in Eq. (8):

$$b_i = \frac{\lambda_i}{C_f + A_i} = \frac{1}{W_i/F_i |W_f| + 1}.$$  

(9)

Here, $|F_i|$ is the number of flows in $F_i$ and $\tilde{W}_f$ is the harmonic mean of the contention windows of the flows in $F_i$.  

Fig. 10. Flow contention graph of flow $i$ and $|F_i|$ independent interferers.

Eq. (9) reveals that when flow $i$ contends only with equivalent flows, $b_i$ does not depend on the REQ packet duration. Eq. (9) also shows that the window ratio $W_i/\tilde{W}_f$ can be used to control the throughput ratio between flow $i$ and its interfering flows. To illustrate application to fairness concepts, we use a simple scenario where all interferers of flow $i$ are not within range of each other. The interference relationships can be represented by a star-shaped flow contention graph centered at flow $i$ (Fig. 10), where vertices correspond to flows and edges correspond to pair-wise interference among flows. If all flows are backlogged, the sum of their success probabilities within each clique of the flow contention graph should equal one.  

Also, in this scenario, the success probability of each flow equals its lower bound. Therefore, $b_f = 1 - b_i$ for every interfering flow $f$ of flow $i$.

It is straightforward to show that a max-min fair allocation in the scenario of Fig. 10 would be $b_i = b_f = 1/2$. This can be achieved by $W_i/\tilde{W}_f = 1/|F_i|$ in Eq. (9). On the other hand, under a proportionally fair allocation, flow $i$ should receive

$$b_i = \frac{1}{|F_i|} \quad \text{and each interferer } f \text{ should receive } b_f = \frac{|F_i|}{|F_i|}.$$  

This can be achieved by $W_i/\tilde{W}_f = 1$ in Eq. (9).

Impact of advantaged interfering flows $A_i$: The expression for $b_i$ considering only the flows in $A_i$ is obtained by setting $F_i = \emptyset$, $D_i = \emptyset$ in Eq. (8):

$$b_i = \frac{\lambda_i e^{-RC_a}}{C_f + \lambda_i} = \frac{e^{2R|A_i|/W_a}}{W_i/A_i |W_a| + 1}.$$  

(10)

where $|A_i|$ is the number of flows in $A_i$ and $\tilde{W}_a$ is the harmonic mean of the contention windows of the flows in $A_i$.

Similar to the case of set $F_i$, Eq. (10) shows that $b_i$ depends on the ratio $W_i/\tilde{W}_f$. However, $b_i$ decreases exponentially with REQ packet duration $R$. The disadvantage due to the REQ packet duration cannot be addressed by flow $i$ decreasing its contention window $W_i$ (the exponential term persists even if $W_i$ is zero). It can only be addressed by increasing $\tilde{W}_a$.

We now rearrange terms in Eq. (10) and express $W_i$ as a function of $\tilde{W}_a$, $R$ and $b_i$:

$$W_i = \frac{\tilde{W}_a}{|A_i| b_i (e^{2R|A_i|/\tilde{W}_a} - b_i)}.$$  

(11)

Eq. (11) gives the contention window pairs $(W_i, \tilde{W}_a)$ that achieve allocation $b_i$ subject to REQ packet duration $R$ and number of interferers $|A_i|$.

Fig. 11. Contention windows yielding max-min fair transmission opportunities for various values of REQ packet size.

Fig. 11 plots Eq. (11) for the scenario of Fig. 10 in the case of max-min fair allocation ($b_i = 1/2$) and $|A_i| = 5$. The curve for $R = 0$ is a straight line with slope $1/|A_i|$; this is also the window ratio that achieves max-min fairness in the case of equivalent flows. For each curve $R > 0$, each point above the $W_i = 0$ line corresponds to a feasible pair $(W_i, \tilde{W}_a)$ that yields max-min fairness. For larger $R$, the required contention window ratio for max-min fairness remains $1/|A_i|$ but the contention window sizes required to achieve it are higher. The above observations make clear that larger $|A_i|$ and $R$ require larger contention windows to ensure a fair number of transmission opportunities among the flows. Larger contention windows require a larger cycle to absorb the overhead due to the contention phase; however, a larger cycle yields higher delays in the network.

C. Impact of clock phase differences

We investigate the impact of clock phase differences to the success probability lower bound $b_i$ predicted by our model. When the relative phases $\theta_{if}$, $\theta_{ia}$, and $\theta_{id}$ are non-zero, the derivation of the closed form expression of Eq. (6) is more tedious than the perfectly synchronized case but can be done using the same procedure as in Section V-B. Here we provide the closed form expression for some special cases of interest. More specifically, we consider the worst case where flow $i$ is lagging all its interfering flows (all phase terms $\theta_{if}$, $\theta_{ia}$, and $\theta_{id}$ are non-negative) and investigate the isolated effect due to each interference set.

Impact of advantaged interfering flows $A_i$: The closed form expression for $b_i$ when only advantaged flows are con-
sidered reads as follows:

\[ b_i = \frac{e^{-2\phi_{ia}(\theta_{ia} + \phi_{ia})}}{W_{ia}^{|A_i|}} \frac{1}{W_a} + 1 \tag{12} \]

where \( \tilde{W}_a(\phi_{ia}) \) is the harmonic mean weighted by the normalized relative phases \( \phi_{ia} \), and \( \Theta_{ia} \) is the average relative phase with respect to flow \( i \).

On comparing with the perfectly synchronized case (Eq. (10)) we observe that the average relative phase adds to the REQ packet duration disadvantage in the exponential term of Eq. (12). In addition, this combined disadvantage in the exponential term can only be addressed by taking into account the contribution of the relative clock phase of each interferer. This can be done by increasing the weighted harmonic mean \( \tilde{W}_a(\phi_{ia}) \) as captured in Eq. (12).

Impact of disadvantaged interfering flows \( D_i \): This is the interesting case where flow \( i \) is advantaged with respect to all interferers yet it lags in phase with respect to all of them. To provide a clear phase advantage to each interferer we assume that flow \( i \) lags by at least \( R \) mini-slots \( (\theta_{id} \geq R) \). The final expression for \( b_i \) in this case, is given by:

\[ b_i = e^{-2\phi_{id}(\Theta_{id} + \phi_{id})} \frac{1}{W_{id}^{|A_d|}} \frac{1}{W_d} + 1 \tag{13} \]

where all quantities are defined similar to Eq. (12). The main observation here is that when the average relative phase \( \Theta_{id} \) equals the REQ packet duration \( R \) the advantage of flow \( i \) due to the REQ packet duration is canceled. At this point the exponential term becomes unity and Eq. (13) becomes Eq. (9)—flow \( i \) competes with its disadvantaged interfering flows as if they were all equivalent in a perfectly synchronized system. However, as \( \Theta_{id} \) increases beyond \( R \), \( b_i \) decreases exponentially with the difference \( \Theta_{id} - R \).

VI. CONCLUSIONS

In this paper, we analyzed the fairness properties of synchronized CSMA network. We showed that in single-hop systems, the throughput of the latest flow decreases exponentially with clock drift, regardless of use of guard time. In multi-hop systems where clock phase differences are coupled with carrier sense and topological bias, we showed that carrier sense in no-guard-time systems can act as a protection mechanism against clock drifts. On the other hand, guard-time systems offer more predictability of throughput. Our model for arbitrary topologies in guard-time systems reveals that the throughput of a topologically disadvantaged flow can decrease exponentially with control packet size and average clock drift of its one-hop interferers. We showed that no-guard-time systems impose tighter requirements on the maximum phase provided by the network clock synchronization mechanism. This is because in a no-guard-time system, the contention window size of each flow must be greater than the phases of all its two-hop neighbors, whereas in a guard-time system, this condition is restricted to one-hop neighbors.

Our derived analytical relationships can provide a foundation for future work that targets achieving network-wide fairness through contention window adjustment. For example, a congestion control algorithm can exploit our results that (i) the exponential disadvantage of a flow due to REQ packet duration and average clock drift can only be addressed by increasing the harmonic mean of the contention windows of its one-hop interferers, and not by decreasing its own contention window; and (ii) increasing the harmonic mean requires very aggressive window increases for each individual interferer.

REFERENCES