High mobility Channel Estimation and Equalization

For OFDM system

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Abstract

The orthogonal frequency division multiplexing (OFDM) is generally used in nowdays high data rate transmission. But, OFDM suffers from interchannel interference (ICI) in rapid variations of doubly selective fading channels even though it is strong at narrow band noise and intersymbol interference (ISI). So, the ICI cancellation should be done at receiver effectively. However, channel estimation and equalization processes in receiver, which mainly works for ICI cancellation, give great complexity for calculating main parameters like inverse of autocovariance of estimated channel. Such calculation is unavoidable to get equalizer tap solutions. In this paper, we introduce low complexity techniques for equalization based on coherent bandwidth and signal power distribution. Those realize lower complexity equalization keeping desirable performances. At last, simulation part will help you to understand.

Keywords: orthogonal frequency-division multiplexing(OFDM), intercarrier interference(ICI)

1. Introduction

The demand for high rate data transmission and the number of mobile device users increases rapidly, high data rate and mobility are the hot topic of recent years. To meet this demand, some techniques were introduced at past, which include reducing the symbol duration or using higher order modulation. In the former method, the signals received through the multipath channel suffer from severe inter symbol interference (ISI) because the delay spread becomes much larger than symbol period. In order to overcome ISI, the symbol duration must be larger than the channel delay spread. In orthogonal frequency division multiplexing (OFDM) system, cyclic prefix extension totally helps to eliminate the effect of ISI, when the guard interval is longer than the maximum length of delay spread. Because of the strength of OFDM to combat ISI, which allows higher data rate transmission, Wireless LAN, Digital Audio and Video Broadcasting use OFDM as the standard.

Despite of these advantages of OFDM technique, however, the increased symbol duration causes two negative effects in a time-varying channel caused by high mobility environment. The change in the channel from symbol to symbol is more significant than the single carrier transmission. Unfortunately, as the speed of mobile station is higher than hundred kilometers per hour, the variation of channel in time domain is faster than symbol rate, called fast fading or time selective fading channel. Time variations of the channel within a multicarrier symbol also lead to a loss of subchannel orthogonality, resulting in interchannel interference (ICI), and it is leading to an irreducible error floor in conventional receiver. So, lots of study about cancellation of ICI has been done.

ICI cancellation technique makes the complexity of receiver be low as the change of channel in time is very fast. The main reasons in calculation of complexity is that the first one is the large number of channels should be estimated, and the last one is the large number of calculation in matrix inversion calculation of MMSE tap solution. Specific value of complexity in receiver is \(O(N^3)\) where N is the number of FFT points, for example, FFT size in DVB is equal to 1512 or 6048. Therefore, complexity issue of receiver is also hot topic in high mobility mobile communication as well as performance issue in that.

This paper is organized as follow. Section 2 describes the basic OFDM system with mathematical expressions and channel model, especially in high mobility environment. In section 3, we talk about the main issues of ICI and next we are arranging the conventional Channel equalization techniques in section 4. Low complexity MMSE equalization algorithm and its simulation result is presented in section 5. Finally conclusions and future tasks are drawn in section 6.
2. System and Channel Model

A. OFDM system block diagram

The OFDM system block diagram is depicted at Fig. 2.1. The FFT and IFFT processor places at center of diagram and added up, because each process almost same except only sign operation. So, like in tranceiver, the system manipulation becomes simpler.

In OFDM transmitter, the binary data get into error control coding block for adding error detecting codes, then it is passing through interleaver to re-ordering binary sequence which gives high endurance from the burst in frequency selective channel. The general used block type interleaver is drawn in Fig 2.2.

The output bit stream from interleaver goes through mapping block which make high data rate, and the output becomes symbol. After that, pilot added symbols for channel estimation, divided to parallel signal, and then IFFT processor gives orthogonality to each subsymbols and parallel to serial (P/S) block put together those subsymbols.

Before to transmit, cyclic prefix (CP) should be added, which helps the received OFDM signal can not be harmed by intersymbol interference(ISI). Guard interval should be larger than the maximum delay profile, $T_{max}$. After adding CP, one OFDM generation is completed.

In receiver, most of blocks do inverse work in transmitter except mainly two parts. First one is synchronization block. Lots of synchronized techniques have been developed in both time and frequency domain. [4, p102–p121] The other is channel estimation and equalization block. Channel estimation uses pilot tones are pre-engaged between transmitter and receiver. The conventional and proposed equalization technique will describe at section 4 and 5.

B. Signal Model

The main concept of OFDM is that the signals are transmitted through $N$ parallel subchannels with keeping orthogonality among the parallel signals. The orthogonality is derived form one of the useful property of exponential function. Fortunately, the main process of OFDM modulator is perfectly same as the IDFT processor, so, the drawback of conventional
multicarrier system which means N oscillator is needed for demodulation, is eliminated by using only one DFT processor. In practice, this transform can be implemented very efficiently by the inverse fast Fourier transform (IFFT). The IFFT drastically reduces the amount of calculations by exploiting the regularity of the operations in IDFT. [2, p35]

Then, suppose the symbol duration after serial-to-parallel conversion is $T_s$; the baseband carrier frequency for the k-th subcarrier is then, $f_k = k / T_s$, $k \in [0, N - 1]$. The entire signal bandwidth is $W = N / T_s$. The transmitted signal over a block, including the cyclic prefix, is given by

$$u(t) = T_s^{1/2} \sum_{k=0}^{N-1} s_k e^{j2\pi f_k t} , -T_p \leq t < T_s \quad (1)$$

where, $s_k$ is the information-bearing symbol on the kth subcarrier, and $T_p$ is the length of the cyclic prefix. Fig.2.3 shows that OFDM spectrum fulfills Nyquist’s criterion for an intersymbol interference free pulse shape, base on equation (1). The pulse shape is present in the frequency domain and it show avoiding by having the maximum of one subcarrier spectrum correspond to zero crossings of all the others.[2, p36]

**Figure 2.3.: The spectrum of individual sub-carriers**

C. Channel model

In most practical wireless communication systems, the radio propagation is far more complex than in a free space situation. In the presence of the earth, natural obstacles, buildings, etc. A radio signal travels via both the direct path and other paths form transmitter to receiver. As a consequence, the radio channel suffers from multipath condition, leading to typical fading phenomena, as well as path loss and shadowing. In other words, the time delays and attenuation factors of the different paths are generally time-varying in mobile communications. Furthermore, we should consider the mobility of mobile station (MS) which causes some tremble in frequency domain called Doppler effect frequency shift.

If we assume the well-known wide sense stationary uncorrelated scattering (WSSUS) model [3, p18–p21], the channel is characterized by its delay power spectrum. If the signal is band-limited, then the time-varying diffuse multipath channel can be represented as a tapped delay line with time-varying coefficients and fixed tap spacing. In this tapped delay line channel duration of the delay power spectrum, or delay spread ($T_d$), and the tap spacing must be equal to or less than the reciprocal of the passband bandwidth. The reason of this is that if that is not accepted, receiver can not estimate channel which includes tapped delay profile. In this paper, we assume that the delay power spectrum profile has a Rayleigh distribution.

If the maximum number of taps is L and the impulse response of channel is $h(n,l)$, then received signal $y(n)$ can be expressed as

$$y(n) = \sum_{l=0}^{L} h(n,l)u(n-l) + w(n) \quad (2)$$

where, $w(n)$ is AWGN. Then, substitute (1) to (2), $y(n)$ can be written by

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k H_k(n)e^{j2\pi nk/N} + w(n) \quad (3)$$

where, $H_k(n) = \sum_{l=0}^{L} h(n,l)e^{-j2\pi kl/N}$, which is the Fourier transform of the channel impulse response at time n, and $0 \leq n \leq N - 1$.

3. Channel estimation and detection

There are two different types of channel parameter estimation: pilot symbol assisted and decision-directed (DD) channel estimation. The former channel estimation for OFDM system uses pilot symbol or trained OFDM symbol. Pilot symbol can be added both time and frequency domain signal and each has own advantages.[4, p93–p98] In [8], pilot has coded to have more endurance and good performance in frequency selective fast fading channel even though it has easy implementation. However, as we mentioned at introduction, in high mobility environment don’t let receiver do channel estimation every OFDM symbol. That means the period of channel changes is shorter than one OFDM symbol period. So, the other channel
estimation is used in high mobility channel rather uses first one. The conventional detection for example Least Square, of multicarrier signals using the FFT exhibits relatively good performance at low values of normalized Doppler frequency, or $f_d T_s$. However, in an environment where the normalized Doppler frequency is high, there is an irreducible error floor, because of destruction of orthogonality among subcarriers. Therefore, the channel estimation and equalization should be performed immediately following the A/D conversion. In this section, we assume that the impulse response of the channel is known at each time and the channel is known.

The received signal $y(n)$ after excluding the guard interval can be expressed in a vector form as

$$y = Hs + w$$

(4)

where

$$y = [y(0), y(1), ..., y(N-1)]^T, \quad s = [s_0, s_1, ..., s_{N-1}]^T, \quad w = [w(0), w(1), ..., w(N-1)]^T$$

and the channel matrix $H$. We can employ the following several detection methods.

\section{Least Square (LS)}

The linear model (4) leads to the classical least squares problem. The least squares detection statistic, $z$ is given by

$$z = (H^H H)^{-1} H^H y$$

(5)

\section{Minimum Mean Squared Error (MMSE)[1]}

The MMSE detector chooses the equalizer matrix $G^H$ which minimizes the cost function $E\{d - z \mid z \mid^2\}$. The resulting detection statistic becomes

$$z = G^H y = H^H (H H^H + \sigma^2 I_p)^{-1} H^H$$

(6)

where $I_N$ is the N-by-N identity matrix. As opposed to least squares, the MMSE detector requires the knowledge of the noise power.

Besides LS and MMSE, decision feedback estimator can make performance higher which successive decision. [6]

\section{ICI Analysis}

\subsection{ICI cancellation}

ICI effect in OFDM system can be derived with dividing received signal into two parts - the former is the portion of original signal and the latter is that of ICI. The derivation of this separation is derived in [7]. So, with iteration process to reduce ICI, The performance of transmission is getting higher than none iteration. The recursive process can be written as follow

$$r^{(i)} = r^{(i)}_1 + h_2^{(i)} (X(i) - \hat{X}^{(i-1)}(i))$$

(7)

where, $r^{(i)}(i)$ is demodulated by taking FFT process and doing I-th iteration. This method also has low complexity in one iteration and after three iterations, the difference the bit error rate (BER) curves between without ICI and after three iterations is very low below 30dB bit energy over noise power.

\subsection{ICI distribution}

As we discussed in 4-A, the symbol in each subcarrier can not deliver its whole energy through its own subchannel due to ICI effect. So, the energy of $s_k$ leaked to the m-th subcarrier can be found as

$$\psi_{m,k} = \varepsilon_s E[|A_{m,k}|^2]$$

(8)

Then the total ICI power on the m-th subcarrier is written as

$$P_{l}^{(m)} = \sum_{k=m}^{N-1} \psi_{m,k}$$

(9)

But the important property of this ICI power distribution, we concentrate is that according to closed Q subcarriers affects very highly. Fig. 4.1 show that this according to $i_d T_s$. As you can see if $i_d T_s$ is equal to 0.9, more than closed six subcarriers (Q=3) has more than 95% of signal power.
This point is the main concept to reduce the number of calculation in MMSE equalization.

\[ y_k = A_k s + w_k \]  
\[ \text{(10)} \]

The MMSE receiver for detecting \( s_k \) based on (10) is \( m_k = R_k^{-1} p_k^* \) where \( R_k = E[y_k y_k^H] \) and \( p_k = A_k (\cdot, k) \). The parameter \( K = 2Q + 1 \) can be chosen to tradeoff between the performance and the complexity. To detect \( N \) symbols, we should find \( N \) MMSE receiver \( m_k \). The major computation involved for each \( k \) is in calculating the covariance matrix \( R_k \) and its inverse. Our techniques reduce complexity are focus on these two calculation. First, the idea is coming from the definition coherent bandwidth which defines a range of high correlation. So, like ICI distribution, when we operate MMSE process, we choose only some portion of \( H \) matrix in (4) derived form coherent bandwidth.

Figure 4.2 shows the overlapped subchannel matrix which is \( L \) by \( L \) square matrix. We also can degenerate non-overlap method, but in that case, subcarriers at edge are estimated by using relatively less correlated channel statistics than other subcarriers, which induces some undesirable error.[9] So we use overlap method.

The complexity comparison of this technique is shown at table 4.1

<table>
<thead>
<tr>
<th></th>
<th>Full MMSE</th>
<th>Overlapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>( N^2 + 2N^2 )</td>
<td>( L^2 + 2(2M-1)L^2 )</td>
</tr>
<tr>
<td>Addition</td>
<td>( 2N(N-1) )</td>
<td>( 2(2M-1)L(L-1) )</td>
</tr>
<tr>
<td>Memory size</td>
<td>( N )</td>
<td>( L )</td>
</tr>
</tbody>
</table>

Table 4.1

The second idea comes from the ICI distribution as we discussed in 4-B. Since the first \( K-1 \) rows of \( R_{k+1} \) are the same as the last \( K-1 \) rows of \( R_k \), we can recursively calculate the inverse of \( R_k \), which greatly reduces complexity.

If we partition \( R_k \) is written as

\[ R_k = \begin{bmatrix} A_k & B_k \\ B_k^H & C_k \end{bmatrix} \]  
\[ \text{(11)} \]

Where \( A_k \) is 1 by 1 matrix and \( B_k \) is 1 by \( N-1 \) matrix, and the rest hole \( N-1 \) by \( N-1 \) matrix is \( C_k \). From the idea of this, \( R_{k+1} \) can be written as follow:

\[ R_{k+1} = \begin{bmatrix} C_k & \tilde{B}_{k+1}^H \\ \tilde{B}_{k+1} & A_{k+1} \end{bmatrix} \]  
\[ \text{(12)} \]
For MMSE solution we use, we need the inverse of $R_k^{-1}$ So, using the notation of (12) we can easily describe $R_{k+1}^{-1}$ which is as follow
\[
R_{k+1}^{-1} = \begin{bmatrix}
C_k^{-1} + b_{k+1}b_{k+1}^Hc_{k+1} & b_{k+1}c_{k+1} \\
 b_{k+1}^Hc_{k+1} & c_{k+1}
\end{bmatrix}
\] (13)
where, $c_{k+1} = (\tilde{A}_{k+1} - B_{k+1}C_k^{-1}B_{k+1}^H)^{-1}$ and $b_{k+1} = -C_k^{-1}B_{k+1}$ and $C_k^{-1}$ derivation is shown at [5].

As we mentioned the major computation to obtain $R_{k+1}^{-1}$ is in calculating $C_k^{-1}$ and $B_{k+1}$ whose complexity respectively $O(N^2)$ and $O(K)$. So, the total computational complexity for detecting an OFDM symbol is $O(N^2K)$. Since $K \ll N$, the computational complexity reduces substantially.

5. Simulation results

As you can see at the Fig. 5.1, channel decomposition method gives only less than 1dB performance loss compare to full MMSE simulation curve even though it makes lower complexity.

![Figure 5.1 performance of channel decomposition method](image)

Also you can check out the second method which using recursive calculation to get $R_{k+1}^{-1}$. Figure 5.2 shows the performance curve when $f_dT_s = 0.05$, $Q=3$ and the modulation scheme is BPSK. Even though $Q$ is equal to 3 whose FFT size is 64, the performance loss is mostly less than 1 dB below 30dB of bit error over noise power. Compared to Figure 5.2 with Figure 5.3, whose $f_dT_s$ value is almost same as 0.1. This means the ratio of Doppler frequency effect become larger. So, overall performance curve becomes higher.

![Figure 5.2 simulation curve (Q=3, fdTs=0.5 BPSK)](image)

And we can compare Fig 5.3 to Fig 5.4 whose $Q$ value becomes zero. That means MMSE processor only consider its own signal power used to its process. We can see the performance of LS curve becomes low and the difference between full MMSE curve and LC MMSE curve is intensively higher.

![Figure 5.3 simulation curve (Q=3, fdTs=0.098 BPSK)](image)
From the performance curves we have gotten show the technique to reduce complexity of receiver can be used effectively while some considerable selections of specific parameters like Q. It is very happy to us these simulation work done well. Through these whole processes of our undergraduate project, we can get closer to lots of issues of mobile channel, and receiver design. If we sum these two low complexity techniques into one system, we expect more complexity becomes low effectively. After this project, if someone do that, proposed system will give low complexity with keeping reliable performance. Lastly, thanks to prof. Park and Kwanghoon, Kim and Yusung Lee who help this project done favorably.

7. References


[9] 노민석, 박현철, “A low complexity LMMSE channel estimation for OFDM system”, Information and Communication University, Daejeon, Korea, 2005