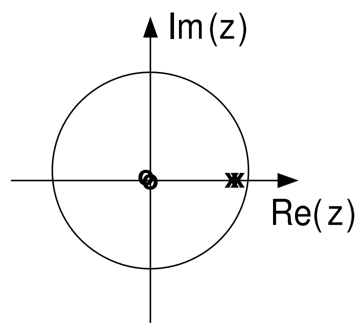
ELEC 431
Digital Signal Processing
Homework 12

Due 5pm, Friday, April 7, 2003

Note: Homework, tests and solutions from previous offerings of this course are off limits, under the honor code.

In this problem you will investigate the design and application of the basic Wiener filter.

a. Suppose that a "smooth" (low frequency dominant) signal $s[n]$ is generated by passing an Gaussian white noise signal $u[n]$ with variance $\sigma_1^2 = 0.01$ through a linear time-invariant filter $A(z)$ whose pole zero plot is depicted below (notice the double poles and zeros). Determine and plot the power spectral density function $S_{ss}(\omega)$ of $s[n]$.



$$A(z) = z^2 / (z-0.95)^2$$

b. Now consider the situation in which we observe a realization of a signal $s[n]$ in additive Gaussian white noise $w[n]$; that is, we measure

$$x[n] \;=\; s[n] \;+\; w[n].$$

Assume that $w[n]$ has variance $\sigma_2^2 = 0.5$ and that $w[n]$ is independent of $u[n]$ (and hence independent of the signal $s[n]$). Design an optimal Wiener filter for estimating $s[n]$ from the measured noisy signal $x[n]$. Give an expression for the Wiener filter $H(\omega)$ in the frequency domain.

c. Implement the signal generation and Wiener filtering process in Matlab.

i. Generate $s[n]$ by applying a time-domain difference equation to a Gaussian white noise sequence $u[n]$.

ii. Generate $x[n]$ by adding another, independent Gaussian white noise sequence $w[n]$ to $s[n]$ (take care to scale the noises to the proper power levels).

iii. Based on the Wiener filter $H(\omega)$ you derived above, devise an FFT-based procedure for filtering $x[n]$. Compare the results of your Wiener filtering to the true signal $s[n]$.

iv. Experiment with different power levels of the noise $w[n]$. How does σ_2^2 affect the output of the Wiener filter? What happens as $\sigma_2^2 \to 0$ and $\sigma_2^2 \to \infty$?