ELEC 431
Digital Signal Processing
Homework 2

Due Monday, January 27, 2002

Note: Homework, tests and solutions from previous offerings of this course are off limits, under the honor code.

1. Recall the EKG (electrocardiogram) example from the lecture notes. Suppose that the clean (noise-free) EKG signal is known to be approximately bandlimited to ± 20 Hz.

a. Is it possible to eliminate the 60 Hz noise without any DT filtering. That is, can the noise be removed using only A/D and D/A converters, possibly with some minor modifications? If so, explain a method for doing this.

b. Is it necessary to sample a rate above 120 Hz ? If not, explain how the noise can be removed even if it is undersampled, and state the minimum sampling rate you think is reasonable.

2. Digital Differentiator:

The goal of this problem is to design a DT filter that mimics the CT differentiation operator. Given an input $x(t)$ the desired output is $\frac{dx(t)}{dt}$.

1. Recall that the differentiation operator is a LTI system. Let $G_c(\Omega)$ denote the frequency response (CTFT) of the differentiation operator. Dervive an expression for $G(\Omega)$. (HINT: Compare the CTFTs of $x(t)$ and $\frac{dx(t)}{dt}$.)

2. Recall that the definition of the derivative is

$$\frac{dx(t)}{dt} = \lim_{h \to \infty} \frac{x(t) - x(t-h)}{h}$$

This suggests the DT approximation

$$\frac{dx(t)}{dt} \approx \frac{\Delta x[n]}{T} \equiv \frac{x[n] - x[n-1]}{T} = \frac{x(nT) - x((n-1)T)}{T}$$

What is the frequency response of the DT filter $g[n] = \frac{\delta[n]-\delta[n-1]}{T}$? Plot the magnitude and phase response of the filter in Matlab.

3. How does magnitude and phase of $G(\omega)$ compare to that of $G_c(\Omega)$?

4. Consider another DT approximation to the differentiation operator:

$$h[n] = \frac{\delta[n+1] - \delta[n-1]}{2T}$$

Compute the frequency response of this filter, plot the magnitude an phase, and compare these characteristics to the desired response $G_c(\Omega)$. This filter's phase response should match the desired phase much better than that of $g[n]$.

5. Suppose $x(t)$ is bandlimited to ± 1 kHz. How fast should we sample in order to guarantee that $h[n]$ reasonably approximates a CT differentiator?