Error Analysis for Digital Communication

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1 Probability of Error

Recall the receiver structure from last class:

\[ r(t) \otimes s(t) \]

\[ \int_{t-T}^{t} r(t) \, dt = nT \]

Choose Largest \( \hat{b}_n \)

![Figure 1: Receiver.](image)

For BPSK, if \( r(t) = s_0(t) \), then the output from the top branch in the receiver is \( A^2T \) and the output from the bottom branch of the receiver is \( -A^2T \). Since the output from the top branch is larger and this output corresponds to \( s_0(t) \), or equivalently \( b_n = 0 \), being transmitted, then \( b_n = 0 \). For \( r(t) = s_1(t) \), we have the output from the top branch is \( -A^2T \) and the output from the bottom branch is \( A^2T \). Again, we choose the correct bit.

For FSK, if \( r(t) = s_0(t) \), the output from the top branch is \( \frac{A^2T}{2} \) and the output from the bottom branch is 0. The opposite will be the case for \( r(t) = s_1(t) \). Here, again, we select the correct bit with our receiver design.

However, communication is usually noisy. How does our receiver perform when noise is present in the received signal? Consider that \( s_1(t) \) is transmitted and the received signal is

\[ r(t) = s_1(t) + N_t. \]

For BPSK, the output from the top branch will be \( -A^2T + N \) and the from the bottom branch we have \( A^2T + N \). Here, \( N \) is a random quantity which is distributed according to the Normal (Gaussian) distribution. Thus, our decision of what bit was sent depends on the realization of \( N \). For FSK, the output from the top branch will be \( \frac{A^2T}{2} + N \) and from the bottom branch will be \( \frac{A^2T}{2} + N \). Since these outputs are closer together, we are more likely to make an error with FSK.

For FSK, the probability of error is

\[ P_e = Q \left( \sqrt{\frac{\int_{0}^{T} (s_0(t) - s_1(t))^2 \, dt}{2N_0}} \right). \]
For BPSK, we have the same formula but we can simplify it further:

\[ P_e = Q \left( \sqrt{\frac{\int_0^T (s_0(t) - s_1(t))^2 dt}{2N_0}} \right), \]

\[ = Q \left( \sqrt{\frac{\int_0^T 4s_0^2(t)dt}{2N_0}} \right), \]

\[ = Q \left( \sqrt{\frac{4A^2T}{2N_0}} \right), \]

\[ = Q \left( \sqrt{\frac{2A^2T}{N_0}} \right). \]

To make the error small, we need to make the argument of the Q-function large. We can do this by making \( T \) large, but this means that \( R = \frac{1}{T} \) is small. Alternatively, we can make \( A \) large, which requires more power.

For FSK,

\[ s_0(t) - s_1(t) = A \left( \sin \left( \frac{2\pi kt}{T} \right) - \sin \left( \frac{2\pi lt}{T} \right) \right), \]

\[ \Rightarrow \int_0^T (s_0(t) - s_1(t))^2 = A^2T. \]

There is more power in BPSK than FSK. Therefore, we conclude that the choice of signal set influences the overall performance of the system.