In the above circuit, there are four nodes: the three black dots and the ground node. The bottom node is considered to be the reference node. We define a voltage for each node in reference to the ground node. In our example,

\[ e_3 = v_{in}. \]

To solve for \( i \), we use KCL to write equations for all the currents exiting the nodes. In this class, it is standard to write the equations for all currents leaving the node, though the actual direction of the currents is arbitrary. The KCL equation for Node 1 is

\[
\frac{e_1 - v_{in}}{1} + \frac{e_1}{1} + \frac{e_1 - e_2}{1} = 0,
\]

\[
\Rightarrow 3e_1 - e_2 = v_{in}.
\]

The KCL equation for Node 2 is

\[
\frac{e_2 - v_{in}}{2} + \frac{e_2 - e_1}{1} + \frac{e_2}{1} = 0,
\]

\[
\Rightarrow \frac{5}{2}e_2 - e_1 = \frac{1}{2}v_{in}.
\]

We combine the two and solve for \( e_2 \).

\[
\frac{13}{2}e_2 = \frac{5}{2}v_{in}.
\]

\[
\Rightarrow e_2 = \frac{5}{13}v_{in}.
\]
Since \( i = \frac{v_2}{R} \),

\[
i = \frac{5}{13} v_{in} A.
\]

The node method applies to general RLC circuit, not just circuits with sources and resistors.

In the above circuit, there are only two nodes besides the ground node. The top left node voltage is \( v_{in} \). We also note that \( e_1 = v_{out} \), the unknown voltage we wish to solve for. We start by writing the KCL equation for the node with voltage \( e_1 \) with all currents exiting the node.

\[
e_1 - v_{in} R_1 + e_1 R_2 + \frac{1}{j2\pi f C} e_1 = 0,
\]

\[
\Rightarrow e_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + j2\pi f C \right) = \frac{v_{in}}{R_1},
\]

\[
\Rightarrow e_1 \left( \frac{R_2 + R_1 + j2\pi f CR_1 R_2}{R_1 R_2} \right) = \frac{v_{in}}{R_1},
\]

\[
\Rightarrow e_1 = \frac{R_2}{R_1 + R_2 + j2\pi f CR_1 R_2} v_{in}.
\]

By definition of the transfer function, we also know

\[
H(f) = \frac{R_2}{R_1 + R_2 + j2\pi f CR_1 R_2},
\]

and since \( H(f) \to 0 \) as \( f \to \infty \), this is a LPF.

2 Conservation of Power

Consider the sum of powers for the elements depicted below. We assume that no currents are entering the loop, the only currents are those drawn: \( i_1, i_2, \) and \( i_3 \). The total power is

\[
\sum_k v_k i_k = (e_a - e_b)i_1 + (e_b - e_c)i_2 + (e_a - e_c)i_3,
\]

\[
= e_a(i_1 + i_3) + e_b(i_2 - i_1) + e_c(-i_2 - i_3),
\]

\[
= 0 \text{ (by KCL)}.
\]

Therefore, power is conserved. Note that we only required KCL and KVL to prove this result. An interesting observation is that if the sum was computed for voltages measured at time \( t_1 \) and currents measured at time \( t_2 \), it is still equal to zero. In this case, it is not total power that is computed, but it is interesting nonetheless.