1 Signals

1.1 Fundamentals

A signal is simply a function, e.g. a function of time $s(t)$. Throughout the course we will often work with signals of the form

$$s(t) = A \sin(2\pi f t + \phi),$$

where $A$ is the amplitude, $f$ is the frequency, and $\phi$ is the phase of the signal. Generally the units of $A$ are voltz ($V$) and the units of $f$ are hertz ($Hz$).

A periodic function is a function which repeats its values in regular intervals (periods).

**Definition:** The period of a function is $T = \frac{1}{f}$, where $f$ is the frequency of the function. This means that for a periodic function, $s(t)$,

$$s(t + kT) = s(t),$$

for any integer $k$.

The function in (1) is a periodic function. These functions are emphasized so much because they are so useful; note that sine waves are used both for communications and to distribute power. Generalizing to complex exponentials, recall Euler’s Formula:

$$\exp\{j2\pi f_0 t\} = \cos(2\pi f_0 t) + j \sin(2\pi f_0 t).$$

From the above formula we can write sine and cosine in terms of the complex exponential.

$$\sin(2\pi f_0 t) = \frac{\exp\{j2\pi f_0 t\} - \exp\{-j2\pi f_0 t\}}{2j},$$

$$\cos(2\pi f_0 t) = \frac{\exp\{j2\pi f_0 t\} + \exp\{-j2\pi f_0 t\}}{2}.$$

1.2 Important Signals

1.2.1 Unit Step Function

$$s(t) = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

The unit step function is discontinuous. The value of the function of $t = 0$ is left undefined.
1.2.2 Pulse Signal

\[ s(t) = p_{\Delta}(t) = \begin{cases} 
1 & 0 < t < \Delta \\
0 & \text{otherwise}
\end{cases} \]

The pulse signal is also discontinuous; the signal is undefined at \( t = 0 \) and \( t = \Delta \). Later we will see that we can decompose the above signal using the unit step function.

2 Systems

A system operates on signals, i.e. it is a function of signals.

The diagram that we use to represent systems is depicted in Figure 2. A example of a system is the Amplifier. In this case, the input signal is simply multiplied by a gain factor, \( G \). Thus, if the input is \( x(t) \), the output is \( y(t) = Gx(t) \). If \( G \) is positive but less than one, the system is called an attenuator; if the gain is negative, it is an inverting amplifier.

Two systems can be cascaded together, meaning that the output of the first system is used an input to a second system. This is shown in Figure 2. Consider two inverting amplifiers cascaded together - this will create a regular amplifier system!
Another important system is the delay system. Whatever the input is, the output is delay in time by $\tau$ seconds. How could we delay a signal? One idea is to add coils of wire for the signal to transverse. Since it takes time for signal to travel (at the speed of light), adding more wire will delay the output of the signal.

When examining signals and systems, you may wish to do signal decomposition, i.e. write the signal as a sum of simpler known signals. For example, for positive $\Delta$, 
\[ p_\Delta(t) = u(t) - u(t - \Delta). \]

Additionally, we should thinking about how feasible certain systems are. Consider a time reversal system, where for the input signal $x(t)$, the output is 
\[ y(t) = x(-t). \]

This system essentially "predicts the future", and is thus unrealizable.

2.1 Linear Systems

Properties:

1. Superposition: Consider inputs $x_1(t)$ and $x_2(t)$ with their respective outputs $y_1(t)$ and $y_2(t)$. For a linear system, if 
\[ x(t) = x_1(t) + x_2(t), \]

then 
\[ y(t) = y_1(t) + y_2(t). \]
2. Scaling: Consider input $x_1(t)$ with associated output $y_1(t)$, and let $a$ be a constant. Then if

$$x(t) = ax_1(t),$$

then

$$y(t) = ay_1(t).$$

Consider the amplifier. It is easily shown that this system is linear. An example of a non-linear system is one that squares the input, i.e.

$$y(t) = (x(t))^2.$$

**Claim:** if the system is linear, then for $x(t) = 0, y(t) = 0$. To prove this, consider the input

$$x(t) = x_1(t) - x_1(t) = 0.$$

By the superposition property,

$$y(t) = y_1(t) - y_1(t) = 0.$$