COMPRESSIVE WIRELESS ARRAYS FOR BEARING ESTIMATION OF SPARSE SOURCES IN ANGLE DOMAIN

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ABSTRACT

Joint processing of sensor array outputs improves the performance of parameter estimation and hypothesis testing problems beyond the sum of the individual sensor processing results. When the sensors have high data sampling rates, arrays are tethered, creating a disadvantage for their deployment and also limiting their aperture size. In this paper, we develop the signal processing algorithms for randomly deployable wireless sensor arrays that are severely constrained in communication bandwidth. We focus on the acoustic bearing estimation problem and show that when the target bearings are modeled as a sparse vector in the angle space, functions of the low dimensional random projections of the microphone signals can be used to determine multiple source bearings as a solution of an ℓ_1-norm minimization problem. Field data results are shown where only 10bits of information is passed from each microphone to estimate multiple target bearings.

***Index Terms*—** Array signal processing, acoustic bearing estimation, compressive sensing, wireless sensor networks

1. INTRODUCTION

Wireless communication technologies have revolutionized the information gathering and processing systems by enabling a large number of simple sensors to coordinate among themselves to tackle larger sensing problems in a bandwidth constrained and distributed manner [1]. In the quintessential application of target localization, the research trend in the sensor networks literature has subsequently shifted from sensor networks of a small number of bearing-capable arrays to sensor networks of large number of proximity-capable sensors. In contrast, recent results in sensor network design suggest that when constrained with the same budget, a sensor network consisting of only arrays can significantly outperform the average localization performance of the cheaper proximity sensors in spite of their sheer number per capita [2].

For arrays, array signal processing is used to enhance the signal-to-noise ratio beyond that of a single sensor's output for parameter estimation [3]. To realize the gains from the joint processing of array outputs, arrays are characteristically tethered since the output data from each sensor in the array generally requires a high bandwidth for transmission. When this transmission is achieved in a wireless setting, the sensor batteries can be quickly depleted and array elements may cause communication interference among themselves as they send relatively large data packets. Compared to wireless proximity sensors, arrays are harder to set up and deploy as they require special deployment mechanisms. In addition, because of their wired nature, arrays tend to have relatively small apertures as unattended ground sensors (UGS), diminishing their main advantage derived from aperture gains. Hence, there is a clear need for a wireless design for arrays to overcome the disadvantages of the tethered array designs to further push the frontiers of what is achievable by sensor networks.

In this paper, we discuss the $2D$ bearing estimation of multiple acoustic sources with a set of sensors using a wireless channel under bandwidth constraints. Typical examples of sources are sniper fire, human footstep and speech signals, vehicle signals, and chirp signals. We employ the recent results in compressive sensing theory, which state that exact recovery of sparse sources may be obtained with high probability from highly under-sampled data in the Nyquist-Shannon sense (see [4] and the references therein). A signal is called *sparse* if it contains only a small number of non-zero components within some transform domain. We demonstrate the feasibility of wireless arrays for bearing estimation when low dimensional random projections of the signals from (possibly randomly) distributed single microphone sensors are used as inter-sensor messages over the communication channel.

We treat the target bearings as a sparse vector in a discretized bearing space and apply ℓ_1-norm minimization with the Dantzig selector [5] as a proxy to a combinatorial optimization problem to obtain multiple source bearings. For acoustic bearing estimation, we assume that the individual sensor locations are known *a priori*; however, the number of sources is not assumed. We explain how the array steering matrix for a sparse set of sources in the angle domain is formed for bearing estimation and how the multiple target bearings are calculated using the random projections of the signals from multiple microphones, which constitute the compressive samples of the target bearings. We note that these projected samples are used directly to calculate the target bearings without any auxiliary signal reconstruction as they may not recover the microphone signals directly. We also give possible implementation schemes for the proposed wireless system. Although we focus on bearing estimation with acoustic signals for acoustic surveillance and teleconferencing, the results can be extended for other types of sources.

Our approach is fundamentally different in many ways from the earlier works for wireless arrays and compressive wireless sensing [6, 7].

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In [6], authors use audio codes and compression techniques to send the full individual sensor acoustic data to a cluster head, which in turn determines source bearings using the maximum likelihood (ML) and least-squares (LS) methods. In [7], power-distortion-latency trade-offs are given for a compressive sensing scheme for sensor networks, which employs the compressive sensing framework as a universal encoding scheme to send and recover signals from multiple distributed sensors.

When compared to [6, 7], our *compressive wireless array* approach provides a wireless sensing strategy to *directly* determine a sparse bearing vector in the angle domain by exploiting the redundancies in the sensor signals for the bearing estimation problem. In our approach, (i) the inter-sensor messages may not recover the original acoustic data sent by a sensor and we do not require any auxiliary signal reconstruction at the processing node in contrast to [6, 7], (ii) we may not be able to determine the source signals even after determining their bearings, and (iii) the inter-sensor messages require significantly smaller communication bandwidth than [6] and smaller bandwidth than the scheme in [7]. We also do not use the ML or LS methods in obtaining our bearing estimates.

The organization of the paper is as follows. Section 2 explains the bearing estimation details of the wireless arrays using compressive sampling ideas. Section 3 gives possible implementation and quantization schemes for message passing among the sensors in the communications channel. Section 4 shows field data results to demonstrate the performance and effectiveness of the wireless arrays.

2. COMMUNICATION CONSTRAINED BEARING ESTIMATION OF SPARSE SOURCES

2.1. Acoustic Data Observations

We discuss the bearing estimation of K noncoherent sources in an isotropic medium in the far field of a collection of M sensors with known positions $\boldsymbol{\zeta}_i = [x_i,\ y_i]'$ ($i = 0, \ldots, M-1$) on the ground plane. The far field of a sensor collection is defined as the boundary of the source region after which the propagating waves appear perceptively planar with respect to the array aperture. For convenience, sensor 0 is called a reference microphone (RM) and is situated at the origin: $\boldsymbol{\zeta}_0 = [0,\ 0]'$. We do not assume that the number of sources K is known.

We denote the received signal at the RM as $x_0(t) = \sum_{k=1}^{K} s_k(t) + n_0(t)$, which is a superposition of K source signals $s_k(t)$ impinging at bearings θ_k (measured with respect to the x-axis) and the sensor noise $n_0(t)$. Sensor i observes the time delayed (or advanced) superposition $x_i(t) = \sum_{k=1}^{K} s_k(t + \tau_i(\theta_k)) + n_i(t)$ of the source signals plus noise, where the time delay at the ith sensor $\tau_i(\theta)$ of a source at bearing θ is given by

$$\tau_i(\theta) = \frac{1}{c}\boldsymbol{\zeta}_i' \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \qquad (1)$$

where c is the speed of sound. Our objective in this paper is to determine the source bearings $\{\theta_1, \ldots, \theta_K\}$ by sending the minimum amount of information possible among the sensors. By determining the minimum information necessary to reconstruct the bearings, we minimize the inter-sensor message packet sizes so that we preserve sensor batteries and minimize inter-sensor communication interference.

2.2. Compressive Sensing

We treat the source bearings $\boldsymbol{\theta}$ as an unknown vector in $\mathbb{R}^N$, where N is the resolution of the (uniformly) discretized bearing space, which resides in $[0, 2\pi)$. An adaptive discretization can be done for focusing purposes. Within the bearing space, the bearings corresponding to the sources have non-zero entries in their respective locations in the discretized bearing space, whose values are to be determined from the problem set up, whereas the zero values simply imply the absence of targets at the corresponding bearings. Hence, our objective source bearings vector is modeled with a K-sparse vector in the N-dimensional angle domain ($N \gg K$), whose *sparsity pattern* is of interest.

Assume that we have digital samples of the source signals corresponding to T seconds, sampled at F_s sampling frequency. Define the kth source vector as a concatenation of these samples:

$$\boldsymbol{s}_k(t_0) = \text{vec}\left\{ s_k(t) \Big|_{t=t_0+\frac{m}{Fs}} ; m = 0, \ldots, \lfloor TFs \rfloor - 1 \right\}, \qquad (2)$$

where t_0 is the time origin and $\lfloor TFs \rfloor > N$. For convenience, we set $t_0 = 0$ for the rest of the paper. Then, if we were to sample the observed signal at a sensor i, we would receive

$$\begin{aligned} \boldsymbol{x}_i &= [\mathbf{0},\ \ldots,\ \mathbf{0},\ \boldsymbol{s}_1(\tau_i(\theta_1)),\ \mathbf{0},\ \ldots,\ \mathbf{0},\ \boldsymbol{s}_K(\tau_i(\theta_K)),\ \mathbf{0},\ \ldots] \\ &\quad \times [\mathbf{0},\ \ldots,\ \mathbf{0},\ 1,\ \mathbf{0},\ \ldots,\ \mathbf{0},\ 1,\ \mathbf{0},\ \ldots]', \\ &= \tilde{\boldsymbol{S}}_i\boldsymbol{\theta} = \boldsymbol{S}_i\boldsymbol{\theta}, \end{aligned} \qquad (3)$$

where $\tilde{\boldsymbol{S}}_i : \lfloor TFs \rfloor \times N$ is the source matrix and $\boldsymbol{\theta} : N \times 1$ is our objective K-sparse bearings vector. Assuming that the sources are noncoherent ($E\{\boldsymbol{s}_k'\boldsymbol{s}_l\} = 0,\ \forall k, l$), we can always replace zero column entries of $\tilde{\boldsymbol{S}}_i$ to make its rank N. Denote one such matrix as $\boldsymbol{S}_i$, where $\text{rank}(\boldsymbol{S}_i) = N$.

Compressed sensing decreases the inefficiency of sampling at F_s by directly acquiring a compressed signal representation without going through an intermediate stage of acquiring $\lfloor TFs \rfloor$ samples [4]. Consider a linear measurement process on the (unobserved) $\boldsymbol{x}_i$ vectors:

$$\boldsymbol{y}_i = \boldsymbol{\phi}_i\boldsymbol{x}_i = \boldsymbol{\phi}_i\boldsymbol{S}_i\boldsymbol{\theta} = \boldsymbol{A}_i(\boldsymbol{\theta})\boldsymbol{\theta}, \qquad (4)$$

where $\boldsymbol{\phi}_i : L \times \lfloor TFs \rfloor$ is the measurement matrix and $\boldsymbol{A}_i(\boldsymbol{\theta}) : L \times N$ is called the source steering matrix. When the source steering matrix satisfies the restricted isometry property (RIP) [5], it is possible to show that $\boldsymbol{\theta}$ can be recovered from $L \geq \alpha K \log \frac{N}{K}$ measurements where α is a small number [4, 5]. However, note that this requires the knowledge of the source matrix $\boldsymbol{S}_i$, which is not known.

2.3. Estimation of Steering Matrices

Estimates of the source steering matrices can be determined using the RM, which is required to take samples at F_s. We form the estimate using the delayed versions of the reference signal as follows:

$$\hat{\boldsymbol{S}}_i(\boldsymbol{\theta}) = \left[\boldsymbol{x}_0\left(\tau_i\left(\frac{2\pi}{N}(0)\right)\right),\ \ldots, \boldsymbol{x}_0\left(\tau_i\left(\frac{2\pi}{N}(N-1)\right)\right) \right]. \qquad (5)$$

Note that when the sought source angle matches the actual source direction, then the columns of the source steering matrix has the maximum correlation, where the other sources act as non-coherent noise samples. When the source steering matrix satisfies the RIP property, it is known that the errors in the sparse vector estimates are well behaved under additive perturbations of the measurements [4]. In [8], we further discuss how each source can be modeled as additive noise in (4) and detail the construction of the steering matrices as a *basis pursuit* strategy.

2.4. Bearing Estimation Problem

Determining $\boldsymbol{\theta}$ has exponential complexity in N as we need to search for all the subsets of N, which is a combinatorial problem. To determine the source bearings, we solve the following convex optimization problem at the RM, which serves as a proxy of the combinatorial solution:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} ||\boldsymbol{\theta}||_1 \text{ such that } \left|\left|\boldsymbol{A}'(\boldsymbol{Y} - \boldsymbol{A}\boldsymbol{\theta})\right|\right|_\infty \leq \epsilon, \qquad (6)$$

where $\boldsymbol{Y} = [\boldsymbol{y}_1', \ldots, \boldsymbol{y}_M']'$, $\boldsymbol{A} = \boldsymbol{\Phi}\hat{\boldsymbol{S}}$, $\boldsymbol{\Phi} = \text{diag}\{\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_M\}$, and $\hat{\boldsymbol{S}} = [\hat{\boldsymbol{S}}_1', \ldots, \hat{\boldsymbol{S}}_M']'$, and ϵ is a relaxation variable. To solve for $\boldsymbol{\theta}$, the RM needs the compressive measurements $\boldsymbol{y}_i$ from the other sensors. Note that the samples $\boldsymbol{y}_i$'s are the compressive samples with respect to $\boldsymbol{\theta}$ and not with respect to $\boldsymbol{x}_i$. That is, it may or may not be possible to reconstruct $\boldsymbol{x}_i$ given the measurements $\boldsymbol{y}_i$. For our bearing estimation problem, we use zero mean Gaussian random variables with unit variance to construct the measurement matrix $\boldsymbol{\Phi}$. To solve for $\boldsymbol{\theta}$, we use the Dantzig selector [9].

3. IMPLEMENTATION DETAILS

We assume that the sensor positions are determined by a calibration algorithm, e.g., [10]. Since the wireless array aperture is expected to be less than 10m for all practical purposes with the number of total microphones not exceeding 10-20, all the communications can be made centralized by using orthogonal coding schemes or can be achieved with a small number of hops, and fairly accurate synchronization can be achieved among the sensors. We assume that a measurement matrix $\boldsymbol{\Phi}$ is predetermined and each sensor has its knowledge.

For the array hardware, we envision a uniform microphone sensor set with wireless communication capabilities, so that each microphone can act as the RM if necessary. With this redundancy, a possible RM bottleneck can be avoided in the future to increase robustness of the system. When a microphone is not acting as the RM, it is in the compressive sensing state to preserve battery and it is called a compressive microphone (CM) in this state. The RM can be chosen randomly; however, it is possible to use heuristics or active self-evaluation methods to choose the best one in some sense. Duties of the RM include: (i) sampling acoustic data $\boldsymbol{x}_0$ at F_s, (ii) forming the sparse source steering matrices (5) using the knowledge of the sensor positions, (iii) receiving messages from the CM's and forming the data vector $\boldsymbol{Y}$ and the measurement matrix $\boldsymbol{\Phi}$, and (iv) determining $\boldsymbol{\theta}$ by solving (6). These duties stipulate a digital embedded system, which can be done with FPGA's or other digital DSP systems.

Full analog, mixed mode, or full digital implementation can be used for the compressive state, depending on the final power consumption of the implementation. In the full analog implementation, analog mixers can be used to simulate $\boldsymbol{\phi}$ to obtain the compressive data $\boldsymbol{y}$ in (4) [11], followed by a simple zero-crossing detector. In this case, the data messages are $\boldsymbol{y} = \pm 1$. Surprisingly, it is still possible to obtain bearing estimates from the solution of (6) (see Experiments). In the mixed mode implementation, an analog-to-digital (A/D) converter is used to sample the analog mixer output. Different quantization levels can be used. In the full digital implementation, the acoustic data is sampled with an A/D converter, then digitally multiplied with $\boldsymbol{\phi}$. Special care must be taken in determining the sampling frequency and the quantization levels for this case.

4. EXPERIMENTS

4.1. Acoustic Field Data Results

A uniform circular acoustic array with 10 microphones (9 microphones on the perimeter with 1.44 meter radius and one at the center) is used to collect the acoustic data for a five vehicle convoy at the Aberdeen Proving Grounds. The acoustic data sampling rate is $F_s = 4410$Hz. The convoy consisted of two military Hummers and three commercial sports utility vehicles, traveling on gravel on an oval track. Detection and tracking of the commercial vehicles presented a difficult challenge because the commercial vehicles were in between the two louder military vehicles, hence they were acoustically suppressed. For this example, we used the center microphone as the RM whereas the other 9 microphones are used as CM's. The array outputs bearing estimates every 0.5 seconds.

Figures 1(c)-(f) illustrate the wireless array bearing estimation results for different number of compressive samples L and quantization levels,

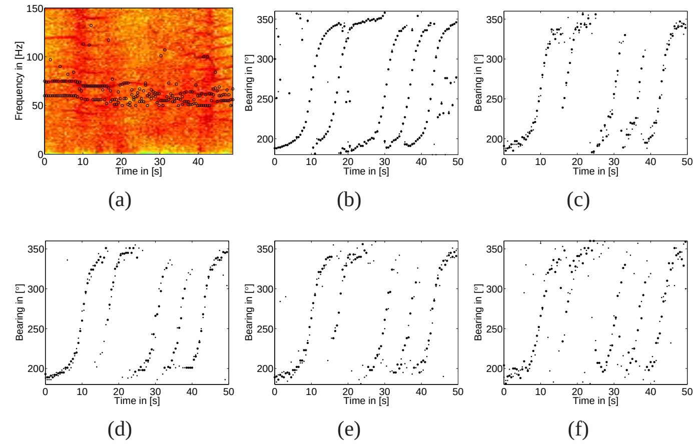

Fig. 1. (a) Time-frequency plot of the received acoustic data is shown. The circles indicate the strongest two frequencies in the data under the spatial aliasing frequency of 150Hz. (b) Minimum variance distortionless response beamforming results on the two strongest frequencies shown in (a). The tracks are smooth with a small amount of clutter. (c) The wireless array results with $L = 15$, each sent with 64bits. The size of the dots is proportional to the value in the solution of $\boldsymbol{\theta}$. There is minimal clutter when compared to the adaptive beamforming results in (b). (d) The wireless array results with $L = 100$, each sent with 1bit (zero crossing). Note that the clutter has increased when compared to the results in (c). (e) The wireless array results with $L = 30$, each sent with 1bit (zero crossing). Note that the clutter has not increased too much when compared to the results in (d). (f) The wireless array results with $L = 10$, each sent with 1bit (zero crossing). The clutter has increased when compared to (d) and (e); however, the bearing tracks are still clear.

and compare them with a frequency adaptive minimum variance distortionless response (MVDR) beamformer (Fig. 1(a)-(b)), which uses all 10-microphone data collected at F_s. The MVDR beamformer uses the strongest two peaks in the time-frequency spectra of the received signal as shown in Fig. 1(a) and detects the three strongest peaks in the power vs. angle pattern. The compressive wireless array uses $N = 360$ and discretizes the bearing space uniformly into 1° resolution grid. For the relaxation variable ϵ in the convex optimization problem, we use $\epsilon = 0.5 \times \sqrt{\log N} = 1.21$ [9]. We report the wireless array results under different test conditions below. In all the cases, a zero mean Gaussian noise with standard deviation 10dB below the power of the compressive samples is also added to the compressive samples *before* quantization.

In Fig. 1 (c), each CM sends 15 compressive samples, each encoded with 64bits. Ignoring the losses of communication overheads and gains of coding schemes, this equates approximately 1000bits/CM for 9 CM's. Figure 1 (d) shows the results when we use the compressive samples $L = 100$ with 1bit quantization (zero crossing). The total communication load in this case is 100bits/CM. When we further decrease the number of compressive samples from $L = 30 \rightarrow 10$ with the same 1bit quantization, corresponding to 30bits/CM$\rightarrow$ 10bits/CM, we see a graceful decrease in the accuracy of bearing estimation and a moderate increase in the clutter. Even with the clutter, the results of the compressive wireless array are quite useful since a random sampling consensus (RANSAC) approach can be used to track the targets [12].

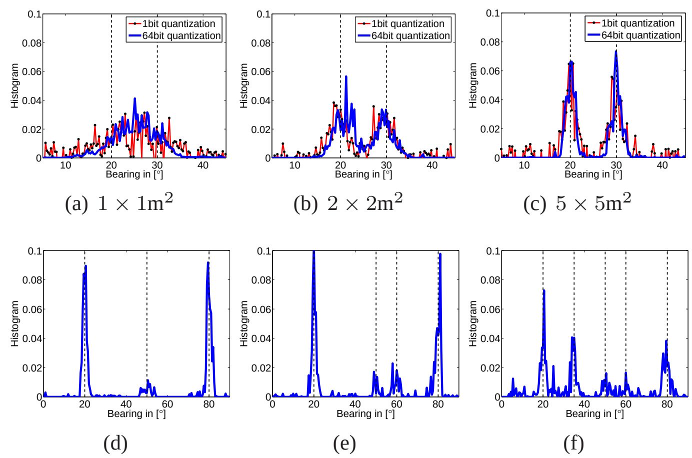

Fig. 2. (*Top*) Aperture gain is illustrated for different aperture sizes. Results with 1bit quantization of the CM outputs are marked with dots. (*Bottom*) Multiple source bearing estimation results for random deployment. The true source bearings are shown with the dashed vertical lines.

4.2. Random Deployment Results

To demonstrate the immediate performance gains with the compressive wireless arrays, we collected vehicle drive-by data for 6 vehicles using a single microphone with $F_s = 4800$Hz. The vehicles and their relevant respective root-mean-squared (RMS) powers for the plots in this section are 1) Nissan Frontier (4.33), 2) Chevy Impala (4.33), 3) Chevy Camaro (4.03), 4) Isuzu Rodeo (2.84), 5) Volkswagen Passat (3.11), and 6) Honda Accord (4.60).

Aperture Gains: To show the aperture gains from the compressive *wireless* arrays, we simulated three scenarios illustrated in Fig. 2(a)-(c). In Fig. 2(a)-(c), we use 9 CMs and randomly deploy them in $1 \times 1\text{m}^2$, $2 \times 2\text{m}^2$, and $5 \times 5\text{m}^2$ aperture, and add an RM at the center. We then use 0.5 seconds of the vehicle data for vehicles #2 and #6 and simulated the array data by placing the sources at 20° and 30° (both at 40m) range by properly delaying each acoustic source by its distance to the microphones ($c = 340$m/s). By using $L = 15$ compressive samples, we determine the sparse $\boldsymbol{\theta}$ for 100 independent Monte Carlo runs, where the individual CM positions vary. We then plot the average of the runs, which creates illustrative histograms seen in Fig. 2(a)-(c). In the figures, we also show results when the estimation is done with 1bit quantization. Similar to the previous section, a zero mean Gaussian noise with standard deviation 10dB below the power of the compressive samples is also added to the compressive samples before quantization.

It is clear that as the aperture size increases with the same number of sensors, the bearing resolution of the arrays increase, allowing the two targets to be separated. This separation is even clear, when only 1bit is used for each compressive sample in spite of the additional clutter. Since the compressive wireless arrays are by design untethered, a random deployment strategy can be used to distribute them over larger apertures than the ones conventionally used for UGS'es. Hence, they are envisioned to perform better than conventional tethered arrays. Finally, it is also interesting to note that the height of the histograms in Fig. 2(b)-(c), which give clues about the relative source RMS powers 4.33 and 4.60, respectively. Also, their shape resembles the Laplacian distribution as opposed to the Gaussian distribution.

Multiple Source Localization: To demonstrate the steering capabilities of our formulation, we simulated three scenarios illustrated in Fig. 2(d)-(f), where we vary the total number of targets from 3 to 5 (all at 40m range). In Fig. 2(d)-(f), we use target configurations $\{\#1, \#3, \#5\}$, $\{\#1, \#3, \#4, \#5\}$, and $\{\#1, \#2, \#3, \#4, \#5\}$, respectively, and plot the $\boldsymbol{\theta}$ histograms for 100 independent Monte Carlo realizations of the random sensor deployment on a $5 \times 5\text{m}^2$ aperture with $L = 15$. The target bearings are given by $\{20^\circ, 35^\circ, 50^\circ, 60^\circ, 80^\circ\}$, respectively. Similar to the previous section, a zero mean Gaussian noise with standard deviation 10dB below the power of the compressive samples is also added to the compressive samples before quantization. As the number of targets increase, there is a gradual increase in clutter peaks; however, the results are still encouraging even at 5 targets that are close in bearing. The height of the histograms seem to be related to the relative source RMS powers.

5. CONCLUSIONS

We have demonstrated the feasibility of a wireless acoustic array to estimate multiple source bearings by passing quantized compressive sensing data among the sensors. In our solution, we exploit the sparsity of the sources in the angle domain and obtain their sparsity pattern, which determines the number of targets and their corresponding bearings. Since the compressive samples are the minimum number of data samples required to reconstruct the bearing vector in the angle domain, our approach use minimum possible communication bandwidth among the sensors. We also showed that there is a significant redundancy in the individual data of the sensors for the acoustic bearing estimation problem. We accomplished this by demonstrating that our wireless array scheme is quite robust against noise in the compressive samples and can even operate when only the zero crossing information of the compressive samples is passed, which cannot be used to recover the data of the sensors. We hope that our results will further improve what is achievable by a wireless sensor networks.

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